Te Poutāhū
Curriculum Centre

Te Mātaiaho

The New Zealand Curriculum

DRAFT

Mātai aho tāhūnui,
Mātai aho tāhūroa,
Hei takapau wānanga
E hora nei.

Lay the kaupapa down
And sustain it,
The learning here
Laid out before us.

Te Tāhuhu o te Mātauranga
Ministry of Education

Te Kāwanatanga o Aotearoa
New Zealand Government

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In the Mathematics and statistics curriculum, the tables in the teaching sequences for phases 1 and 2 are spread across two pages. Therefore, on screen, the curriculum is best viewed using the 'double pages' document, so that each double page spread is visible.

If you are printing the document to view it, print the 'single pages' version, backed. Then staple or bulldog clip the left-hand side of the printout, so that as you go through the document, each double spread is visible when you turn the pages.

The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The New Zealand curriculum is a **knowledge-rich** curriculum that prioritises and explicitly describes what students are expected to understand, know, and be able to do in each year of schooling. It is deliberately sequenced to enable students to build knowledge, skills, and competencies systematically over time.

The curriculum framework of Te Mātaiaho has been developed by successive governments, demonstrating a commitment to longevity, stability, and enduring support for a world-leading New Zealand curriculum.

It is through the professional capability, experience, innovations, contexts, and skills of Aotearoa New Zealand's teachers and educators that this document will be brought to full life in the classroom, through local, national, and global contexts.

The **science of learning** informs the curriculum's sequencing by building on an understanding of how we learn:

- We learn best when we feel a **sense of belonging** in our learning environments and feel **valued and supported**. Students bring with them different cultural identities, knowledge, belief systems, and experiences. They need to see that these are valued and reflected in a school environment characterised by mutual respect and sensitivity towards students' individual needs, emotions, cultures, and beliefs.
- A new idea or concept is always interpreted through, and learned in association with, **existing knowledge**. Therefore, the amount of existing knowledge a student has about a topic and the degree to which it is interconnected influence both the quality and ease of learning. Strong relationships between teachers and students and the recognition of students' prior knowledge are connected to improved learning outcomes.
- **Emotions** directly impact our ability to engage in learning. Positive emotions can enhance the process of learning new knowledge and skills and lead to deeper, more permanent learning for students. Conversely, negative emotions can inhibit a student's ability to learn.
- **Motivation** is critical for engagement in learning and wellbeing. Success in learning also helps to build motivation. Motivation is promoted when students feel that three basic needs are met – autonomy, competence, and connection.

The whakapapa of Te Mātaiaho draws together these themes in a structure that is coherent and inclusive for all students.

The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The design of this framework encompasses seven curriculum components. Te Mātaiaho as a whole is about weaving together these components, all of which begin with the word 'mātai', meaning to observe, examine, and deliberately consider.

Mātaiahikā | Relationships with tangata whenua and local community

Learning through relationships with tangata whenua and local communities

Mātai kōrero ahiahi. | Keep the hearth occupied, maintain the stories by firelight.

Poutama curves represent relationships with tangata whenua and the community.

Mātaioho | National curriculum – contextualised

The process by which schools bring the national curriculum to life through local, national, and global contexts

Mātai oho, mātai ara, whītiki, whakatika. | Awaken, arise, and prepare for action.

Unaunahi scales represent wealth of knowledge, purpose, and know-how.

Mātaiaho | Learning areas

The eight learning areas, which each include a purpose, big ideas, knowledge, and practices, year-by-year

Mātai rangaranga te aho tū, te aho pae. | Weave the learning strands together.

Taratara-a-kae niho notches represent diversity, resilience, and mana.

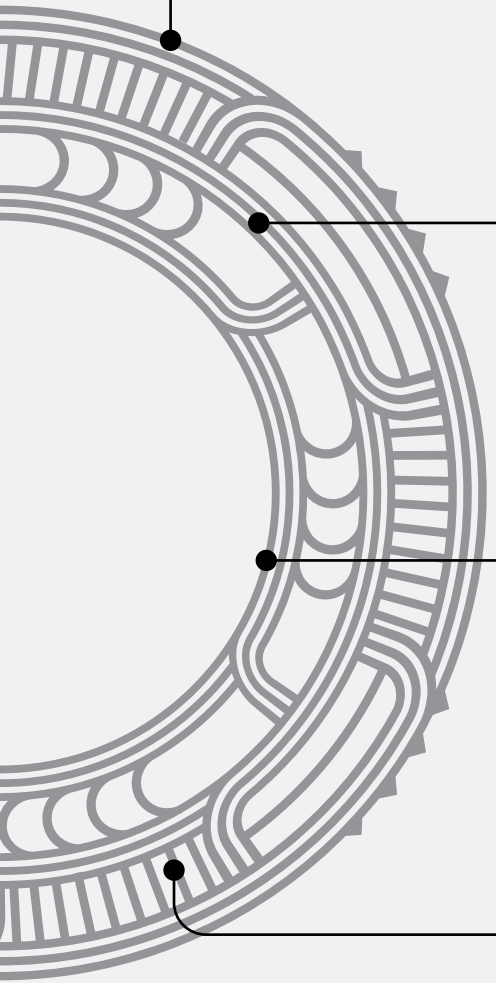
› **Mathematics and statistics years 0–8**

Mātairangi | The guiding kaupapa

The overarching kaupapa guiding the curriculum, based on the science of learning and ensuring excellent and equitable outcomes for students

Mātai ki te rangi, homai te kauhau wānanga ki uta, ka whiti he ora. | Look beyond the horizon, and draw near the bodies of knowledge that will take us into the future.

The outer rings represent our guiding kaupapa.



Mātainuku | Creating a foundation

The curriculum principles (e.g., holding high expectations, and enabling all students to access the full scope of the curriculum)

Mātai ki te whenua, ka tiritiria, ka poupoua. | Ground and nurture the learning.

The centre rings represent the foundation and calls to action.

Mātaitipu | Vision of young people

The educational vision of young people, as conceived by young people

Mātaitipu hei papa whenuakura. | Grow and nourish a thriving community.

The inner rings and circular space represent the vision and students at the centre.

Mātairea | Supporting progress

The whole schooling pathway and the overarching focus for year-by-year learning and progress

Mātai ka rea, ka pihi hei māhuri. | Build and support progress.

Niho kurī lines represent building and supporting the development of students.

DRAFT

Te Mātaiaho

The New Zealand Curriculum

MATHEMATICS AND STATISTICS YEARS 0–8

Purpose statement

Ānō me he whare pūngāwerewere.
Behold, it is like the web of a spider.

In the mathematics and statistics learning area, students study, learn, and appreciate the power and beauty of abstraction, reasoning, and symbolic representation. They learn to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. Mastering mathematical concepts means students can accurately and efficiently use mathematics and statistics (maths) as a foundation for new learning and to solve problems, reason, and make connections between concepts. The above whakataukī points towards the connections between different concepts, knowledge, and processes that are central to mathematics and statistics.

Students discover personal enjoyment and satisfaction in the learning area as a result of engaging teaching methods that build confidence through incremental challenges and that promote curiosity through the exploration of maths concepts. They appreciate that maths provides tools for powerful problem solving and discover how these tools can improve their lives and the lives of others.

Mathematics and statistics often serve as a universal language fostering collaboration, innovation, and mutual understanding. Students participate when they engage in discussions about their maths thinking and the thinking of others. They also discuss important social matters such as the ethical gathering, interpretation, and communication of data, and challenging misinformation and disinformation.

Mathematics and statistics have a history that involves many cultures contributing to innovation and shaping our thinking today. Students come to understand the value of mathematical modelling and statistical investigation as a lens for resolving collective global challenges.

Through the learning area, students develop fluency and mastery in maths, which leads to pathways into a wide range of industries that rely on maths knowledge and reasoning. Fluency and mastery also allow students to participate fully in an increasingly technology- and information-rich world of work.

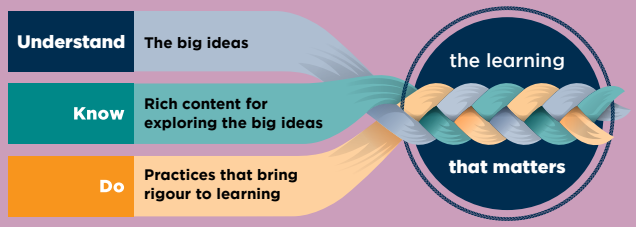
Learning in maths builds both literacy and numeracy. Maths contributes to students' literacy by developing their skills in oral and written communication, reasoning, and comprehension. The learning area supports the use of specific vocabulary and symbols and the understanding of tables, graphs, and diagrams, as well as critical thinking about the quality of data and stories told about it.

The learning area embodies a structured approach to mathematics that encompasses:

- a clearly sequenced year-by-year curriculum
- a progressive and cumulative approach to the acquisition of knowledge, skills, and competencies in order to build student mastery
- teaching guidance on effective practice.

The learning area will be supported by high-quality, curriculum-aligned instructional resources.

Understand–Know–Do Overview



Understand
Big ideas

Understand describes the deep and enduring big ideas and themes that students develop over phases 1–5.

UNDER DEVELOPMENT

The Understand-Know-Do Overview will be completed when all five learning phases have been completed.

Know
Content and concepts

Know comprises the meaningful and important content, concepts, and topics that exemplify and enrich students' understanding of the big ideas.

UNDER DEVELOPMENT

The Understand-Know-Do Overview will be completed when all five learning phases have been completed.

Do
Practices

Do describes the practices (skills, strategies, processes, and competencies) that are particular to each learning area and that bring rigour to learning.

UNDER DEVELOPMENT

The Understand-Know-Do Overview will be completed when all five learning phases have been completed.

Phases

1 Years 0–3

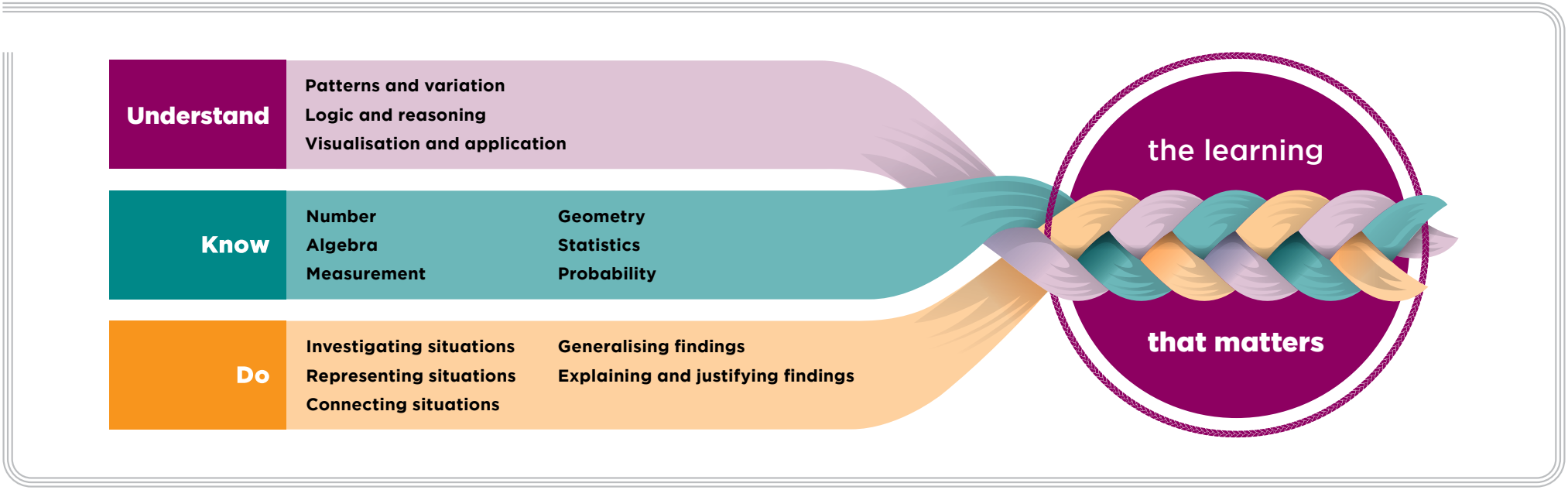
2 Years 4–6

3 Years 7–8

Mathematics and statistics learning area structure

As the Overview on the preceding pages shows, there are three elements in the mathematics and statistics learning area: Understand, Know, and Do.

- **Understand** helps connect school maths with the wider world and identifies critical learning in maths. Understand comes alive as students engage in the practices of Do to learn the knowledge embedded in Know.
- **Know** is broken down into six strands that represent the key learning in maths: number, algebra, measurement, geometry, statistics, and probability.
- **Do** includes processes that provide ways of presenting, applying, investigating, manipulating, and connecting the mathematical knowledge in the curriculum. These processes are central to how students learn and apply maths knowledge.



The year-by-year teaching sequences are organised in line with the strands of Know. The statements that begin with '*Use the mathematical processes to:*' describe explicit teaching for the Do practices that supports the learning of key concepts, procedures, and skills. They Intentionally incorporate literacy skills through the use of verbal representations, vocabulary, word problems, and student explanations and justifications to enrich the learning.

Each statement in a sequence varies in the amount of teaching time it requires. As an overall guide, the weighting for each strand and sub-strand is indicated by the total number of statements. Some statements are repeated across multiple years, allowing more time for progression, consolidation, and emphasis in other areas.

The teaching methods within the teaching sequences describe recommended teaching representations or approaches for specific statements. They support students to develop fluency, to apply their knowledge in creative ways and in varied contexts, and to problem solve.

The science of learning

The science of learning provides insights into how to sequentially structure teaching and learning programmes to ensure progressive mastery of maths concepts. It also provides pedagogical principles that support student learning in maths.

Maths learning is cumulative in nature, with each new concept building on others that have already been learned. Therefore, students' mastery of concepts is critical for ongoing learning and success in maths. Mastery in maths combines both conceptual and procedural knowledge. It is important, therefore, that students comprehend what they are doing and why they are doing it (a conceptual focus) as well as developing fluency and automaticity (a procedural focus). This enables them to accurately and efficiently use their maths knowledge as a foundation for new learning.

Learning in maths imposes a substantial cognitive load. Cognitive load is reduced when teachers break down learning into small steps, making sure that each step is mastered and automaticity is achieved before progressing.

Mastery of maths is supported by pedagogical approaches that use concrete and pictorial representations to introduce new maths concepts, before moving on to abstract representations. Students also require repeated opportunities to practise new knowledge and skills in different contexts and through a variety of tasks. They need timely feedback to correct any misconceptions before those misconceptions are encoded in long-term memory, at which point they are much harder to correct.

As students progress in their maths learning and become more procedurally fluent, they should be given opportunities to apply the concepts and procedures they have mastered. Applying knowldge to new situations, and engaging in more complex tasks such as problem-solving activities, requires and develops higher-order thinking skills.

Phases

1 Years 0–3

2 Years 4–6

3 Years 7–8

Teaching guidance

A comprehensive learning programme

A comprehensive mathematics and statistics programme has the following elements:

1. Positive relationships with maths

To support students in developing a positive relationship with maths, focus on building their confidence by setting high, positive expectations and ensuring every student experiences success. Engage with students' interests outside the classroom to create meaningful learning experiences, and reinforce that maths knowledge, skills, and processes can develop with consistent effort. Provide manageable challenges and teach perseverance, valuing mistakes as learning opportunities. Encourage risk-taking and exploration of maths in real-world contexts, such as designing a school garden or collecting data for decision making.

Create an inclusive learning environment where all students feel supported to contribute and take risks. Teach active listening, reflection, and collaboration skills, and connect maths tasks to students' languages and cultures. Work with students' families, inviting them to be involved in everyday maths activities and classroom maths work.

Model personal engagement in maths processes, so that you build knowledge, curiosity, and enjoyment in maths while teaching.

2. Maths mastery and a structured approach to teaching and learning

Maths mastery is a teaching approach that builds deep conceptual understandings by carefully sequencing mathematics and statistics content. Students are supported to master content through the use of material and pictorial representations, deliberately designed consolidation, and opportunities to apply concepts to new situations. When students have mastered a maths concept, they can accurately and efficiently use it as a foundation for new learning and to solve problems, reason, and make connections between concepts.

When planning and teaching maths units and lessons using the maths mastery approach, it is important to consider the purpose of learning and to include a balance of practices across the week.

3. Effective use of technology

Carefully chosen technology can enhance the teaching and learning of maths. Such technology includes digital applications, calculators, and software. Plan to use technology to make maths engaging and accessible for students, supporting their understanding of concepts.

Where appropriate, use technology to:

- represent and explore patterns and functions
- draw graphs and data visualisations
- perform calculations
- visualise shapes, space, and transformations
- collect, analyse, and interpret data
- organise and communicate findings and reasoning.

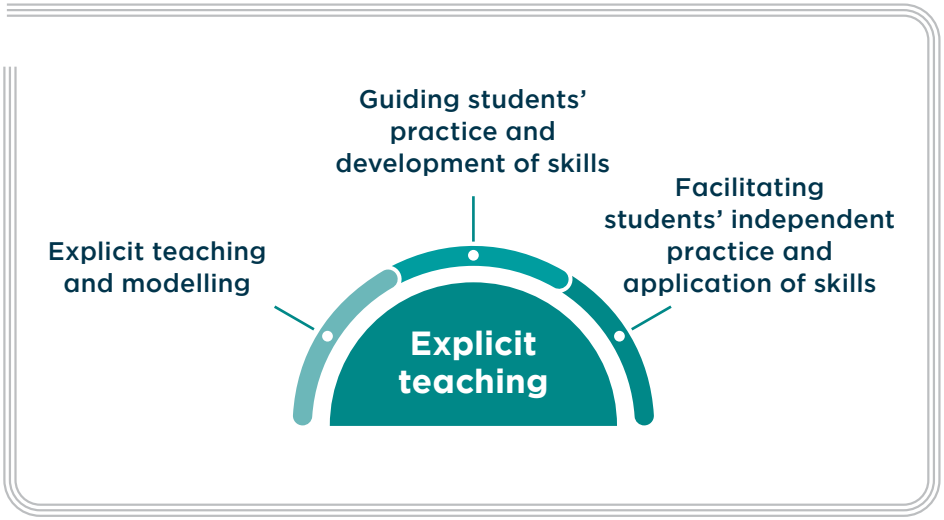
Students should be taught to use technology accurately, appropriately, and efficiently as a tool to support their purpose. This is an important skill in maths. However, this does not mean that the use of technology replaces drawn diagrams or written and mental calculations. Students need the ability to estimate and reason to ensure they can evaluate whether findings generated by technology are reasonable and effective.

4. Explicit teaching

Intentional teaching is purposeful and planned. It involves teachers constantly noticing, recognising, and responding to students' learning to make sure that they are achieving the learning purposes.

Explicit teaching is an important aspect of intentional teaching. It means that teachers understand and explain concepts thoroughly and clearly, with planned and defined purposes for each lesson. They do not expect students to work out these concepts naturally or intuitively on their own.

Explicit teaching is a valuable approach when learning is new, and when misunderstandings and gaps have been identified. It is not all about teacher talk. Instead, teachers gradually hand over to students the responsibility for their own learning. Beginning with teaching and modelling, teachers move to guided practice and skill development, and finally to facilitating students' independent practice. Students are active learners throughout this process. This process is repeated as students encounter new learning.



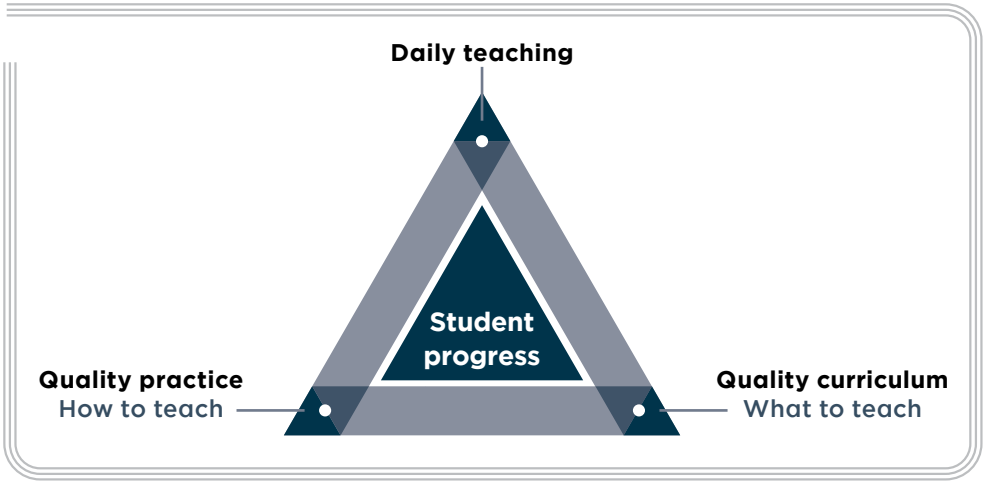
Planning

Teaching and learning plans are developed for a syndicate, year, topic or unit, week, and lesson, and differentiated to meet the needs of individual students. The following considerations are critical when planning.

- Design organised and focused plans in order to make optimal use of instructional time.
- Identify the critical content that will provide rigorous learning for all students by combining teaching sequence statements from across strands (e.g., statements from number, geometry, and measurement).
- Sequence the knowledge and practices in ways that take into account such things as easier and harder aspects, highly frequent to less frequent usage, prerequisites, and associated knowledge and practices.
- Break down complex skills, strategies, and knowledge into smaller instructional units.
- Use assessment information to identify particular learning and teaching strategies that reflect students' prior knowledge and that will extend and deepen this knowledge. Provide learner-focused feedback and reflect on and evaluate students' responses to learning to plan future teaching.
- Use flexible groups within a lesson, based on the purpose of learning for the lesson (e.g., working as a whole class to demonstrate and discuss, before moving into smaller groups to investigate a situation or solve a problem).

An hour a day of maths

Students progress best when they have daily opportunities to learn through evidence-informed, high-quality approaches and teaching practices.



Teach maths for an average of an hour a day, including a dedicated lesson using the structure in the following table. Integrate maths instruction across a range of learning areas, and, if appropriate, spread the hour of dedicated instruction across the day to maximise students' engagement. Space the learning of a skill, concept, or process over several lessons so that students have multiple opportunities to learn and to consolidate their learning by applying it to new topics.

Structure for dedicated maths lessons

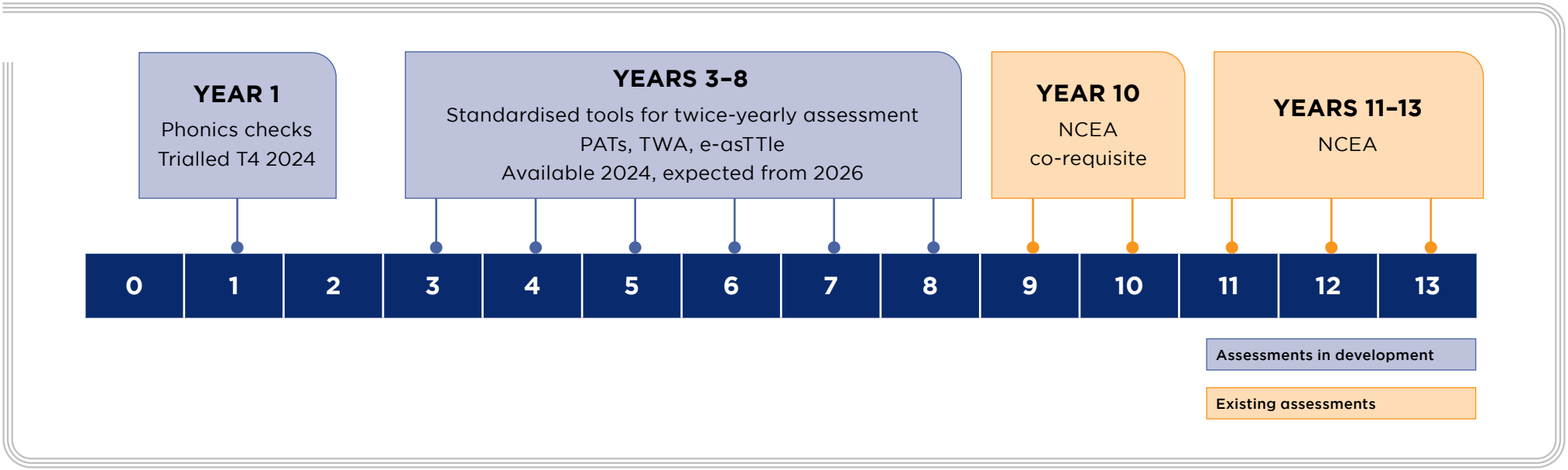
A 'getting started' introduction	Plan to: › link to prior learning to ensure all students can access and understand new concepts or processes › check students' understanding as you extend previous learning › introduce new concepts using a focus activity, group challenge, problem solving, or a task that activates prior knowledge and interests.
Working time Depending on the purpose for the lesson, aim to include one or more experiences. As students are working, take time to notice, recognise, and respond to their learning.	Plan to: › explicitly teach, leading back and forth interactions and including explanations, demonstrations, questioning, short tasks, and discussion, to enable students to think, reason, and apply their knowledge to solve problems › provide whole-class, small-group, paired, or individual work opportunities where students develop or consolidate concepts and practices through investigations, problem solving, tasks, or games › provide additional explicit teaching based on the learning needs of individual students › help students organise new knowledge in ways that connect with their prior knowledge › provide independent or guided practice and activities supporting active retrieval › use review activities to consolidate learning and inform next steps.
A 'connect and reflect' summary	Plan to summarise the lesson: › making clear connections back to the purpose of the lesson › reviewing learning by discussing, sharing, and analysing learning pathways and findings › taking pre-teaching opportunities to prime students for the next lesson. Collect information about what was learnt through: › self and peer assessment, with students reflecting on goals and identifying next steps › checking conceptual understanding, with an emphasis on procedures, vocabulary, and representations › noticing and highlighting curiosity, resilience, perseverance through challenges, and progress.

Use of standardised assessment tools

Schools are expected to use the assessment tools shown in the table below so that we have a nationally consistent approach to assessment and a common set of assessment tools. Teachers determine when the assessments take place.

The tools reflect the shifts in the focus of mathematics and statistics learning throughout the schooling pathway. Assessment information from the tools will help teachers to make decisions about responding to the learning needs of students. This will help school leaders to prioritise and monitor urgent action when it is required to support classroom teaching. Where teaching needs to be targeted and intensified to meet specific needs for finite periods, leaders can draw on a breadth of available supports as required.

Comprehensive system for assessment and aromatawai from 2025



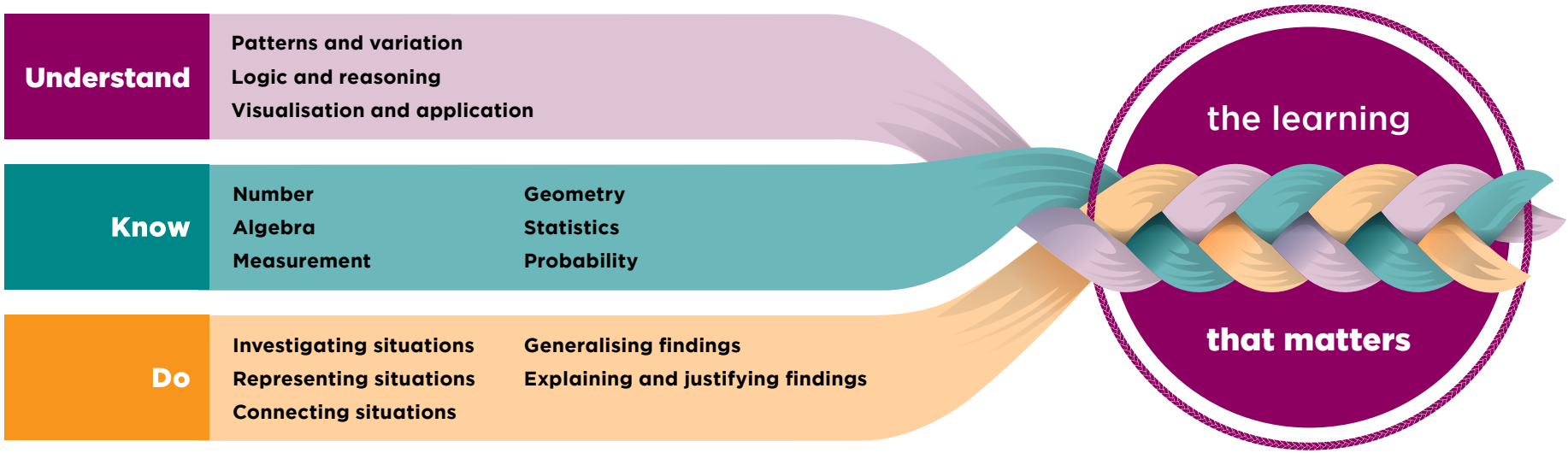
Phase 1

Years 0–3

Thriving in environments rich in literacy and numeracy
Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau

Progress outcome by the end of year 3 (Foundation)

In phase 1, students weave together Understand, Know, and Do and use logic and reasoning to investigate, classify, and describe patterns and variations in quantities, shapes, and data. They use materials, number lines, and pictures to visualise these concepts, make connections between representations, and explain their reasoning. They begin to generalise and to understand the properties of numbers and shapes.



Understand
Big ideas

As they develop their maths capabilities and knowledge, students come to understand that the world has structures useful for noticing, exploring, and describing **variation** and different types of **patterns** and relationships. Students are developing **logical** reasoning and critical thinking skills that enable them to seek information, ask questions, and share their mathematical reasoning.

Statistical **reasoning** allows them to begin to explore their immediate world and make statements about what they observe. Students are developing the ability to **visualise, which allows them** to understand and interact with maths ideas. They **apply** these ideas to explore everyday situations and to support decision making and the communication of ideas.

Know
Content and concepts

Number | Mātauranga tau

By the end of this phase, students know that our **number system is base 10** with ten digit symbols. The place value of a digit in a number depends on its position; as we move to the left, each column is worth ten times more. Zero is used as a placeholder. Students know that they can use subitising patterns to support **estimations** and calculations. They know that numbers can be **partitioned and recombined** in different ways. Addition is putting parts together to find the total or whole. Subtraction takes parts away from the whole; it is also the difference or distance between numbers. Multiplication and division involve recognising and working with **equal groups**: how many are in each group, the number of groups, and the total amount in all groups.

Students know that **fractions are numbers** that can be described using words, pictures, or symbols. When fractions are represented symbolically, the bottom number (denominator) shows how many pieces a whole has been equally split into, and the top number (numerator) shows how many of those parts the fraction represents. The bigger the denominator of a fraction, the smaller the pieces. **Fractions show parts of a whole** region, set of objects, or measurement; they also show the division of two numbers (the quotient).

Algebra | Taurangi

Students come to know that committing **maths facts** to memory allows them to be recalled fluently, so that attention is freed for working on more complex tasks. The **equal sign is relational** in that it shows that the two sides of an equation represent the same quantity. Students notice properties in basic operations: **inverse operations** undo each other (addition and subtraction, multiplication and division), the **commutative property** applies to addition and multiplication, the additive **identity** is 0, and the multiplicative **identity** is 1.

Students know that **patterns** can be made of elements, including numeric or spatial elements, in a sequence governed by a rule. Repeating patterns have a unit of repeat; growing patterns can increase or decrease. Students also know that an **algorithm** is a set of step-by-step instructions to complete a task or solve a problem.

Measurement | Ine

By the end of this phase, students know that **systems of measurement** have a history and that different cultures use different approaches to measuring. Students know that they can **measure and compare various attributes**, such as length, area, volume, capacity, mass (weight), temperature, duration, and turn, using informal or standard units. When measuring, the measurement units must remain the same and join up continuously with no gaps or overlaps. The measurement is the total number of units used from start to end. Students also know that the distance around the edge of a two-dimensional shape gives perimeter, covering a surface gives area, and filling a three-dimensional shape gives capacity or volume.

Geometry | Āhuahanga

During this phase, students come to know that **patterns and regularities in shapes** can be used to compare, classify, and predict. Two- and three-dimensional shapes have **features** that can be observed and described using **geometric language**. Shapes and objects can flip (reflect), turn (rotate), slide (translate), and be used to create patterns. Objects can be rotated in space and may appear different from other **perspectives**. Students know that **maps** are two-dimensional representations of places in the world with symbols to show locations and landmarks. The **position** of a location can be described relative to another location, including a known environmental feature.

Statistics | Tauanga

By the end of this phase, students know that **data** is information about the world, that it comes in many forms, and that it helps them to learn about people, their lives, and their environment. They know that a **statistical enquiry cycle** can be used to investigate a group using questions that they ask of the data. A variable refers to an attribute of the data, such as height, number of children, or colour. **Sorting and organising** variables helps to make sense of data and to answer **summary investigative** questions. **Data visualisations** are representations of all available values of one or more variables that reveal relationships or tell a story.

Probability | Tūponotanga

Students come to know that a chance-based situation has a set of possible outcomes that can be arranged into events. The probability of an event is the chance of it occurring.

Do
Practices

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can work with others to pose a question for investigation, find entry points for addressing a question, plan an investigation pathway, and follow it. They can identify relationships and relevant prior experience and knowledge to support the investigation. They can describe progress on the investigation pathway and work with others to make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

Students can use representations to explore, find, and illustrate patterns. They use representations to learn new ideas and explain ideas to others, and they use visualisations to mentally represent and manipulate groups and shapes. They select or create appropriate mental, oral, physical, or virtual representations.

Connecting situations | Te tūhono pūāhua

Students can suggest connections between ideas, approaches, and different representations. They connect new ideas to things they already know. They make connections with ideas in other learning areas and with familiar local contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

Students can recognise and explore patterns, and make conjectures and draw conclusions about them. They can identify relationships, including similarities, differences, and new connections. They look for patterns and regularities that might be applied in another situation or always be true. They make and test conjectures, using reasoning and counterexamples to decide if they are true or not. They use words and pictures to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

Students can make statements, give explanations about what they notice and wonder, and make deductions based on prior knowledge. They ask questions to clarify and understand others' thinking and use evidence and reasoning to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others, and they present basic explanations and arguments for an idea, solution, or process.

Phase 1
Years 0–3

Teaching sequence

Thriving in environments rich in literacy and numeracy
Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau

Throughout phase 1, students should experience teaching that encourages curiosity and fosters success, as they explore environments and contexts rich in number and spatial elements. Students engage in active, hands-on experiences that involve them doing maths through meaningful tasks that engage their interests and reflect the world outside the classroom.

Continuously monitor students' reasoning, questions, representations, and use of materials, and act quickly to respond to any misconceptions. Ensure your teaching builds on what students already understand, know, and can do.

Throughout each day, provide opportunities for students to:

- **develop positive relationships with maths –** plan meaningful problem solving and investigations into familiar situations, as well as the exploration of rich mathematical situations
- **'have a go' and take risks –** reinforce the idea that mistakes help us learn as we try new skills or contribute their ideas
- **experience explicit teaching of concepts and practices –** use worked examples with new learning broken down into clearly explained, manageable steps
- **listen to the teacher 'think out loud' –** voice your thinking to show your decision making about which numbers or operations to use as you demonstrate a skill or process

- **use numbers, materials, and pictorial representations (diagrams, pictures) –** select representations that support the purpose of learning and that will help students represent their thinking and reasoning and learn new ideas. Over the phase, move students towards using symbols and showing operations as number equations. Number lines are a key representation in this phase for representing, ordering, and comparing numbers (including fractions) and demonstrating operations
- **build vocabulary –** use games, songs, word walls, books, and online tools, and be intentional about the vocabulary you use. This supports students to connect their informal language to reasoning, using maths vocabulary. Draw on your students' first and heritage languages
- **engage in the maths practices to connect learning within and across contexts –** point out connections within concepts (e.g., "If I know 3 + 4, then I know 4 + 3), and teach connected statements together (e.g., quarters, quarter turns, and quarter hours). Encourage students to connect with maths outside school by bringing in photos resources, books, and cultural artefacts from home that link to maths learning
- **visualise and identify patterns, connections, and structures in shapes and numbers –** engage students in sorting, classifying, grouping, and partitioning, discussing what they have noticed and what they are wondering about
- **engage with you and their peers –** support interactions that allow students to discuss, explain their maths ideas, summarise, ask questions, and make suggestions. Over phase one, help students to recall and connect maths learning using questions, materials, and verbal or visual prompts
- **consolidate maths learning to develop fluency with time-sensitive statements in the teaching sequence –** plan a range of tasks, such as using songs, families of facts, games, and apps to practise skip counting, addition, subtraction, multiplication, and division facts.

The highlighted cells in the teaching sequence show critical steps that alert teachers to specific aspects of learning that are essential and time-sensitive. They support teachers to notice, recognise, and respond to student learning in a timely way.

Number

	6 months *Informed by formative assessment, teach students to:*	**Year 1** *Informed by formative assessment, teach students to:*	**Year 2** *Informed by formative assessment, teach students to:*
Number structure	› subitise (recognise instantly) the number of objects in a group of up to 5	› subitise (recognise instantly) the number of objects in a group of up to 10 objects, including combining two patterns of 1–5 objects	› group objects in a pattern of at least 10 objects, subitise the number of objects in each part, and find the total number in the pattern using the parts
	› count to 10 and beyond to 20, forwards and backwards, from any number	› count to 20 and beyond to 100, forwards and backwards in 1s, 2s, and 10s, from any number	› count to 100, forwards and backwards, from any number, in 1s, 2s, 5s, and 10s
		› recognise and represent the ten-and-ones structure of the 'teen' numbers 11–19	› recognise and represent the base ten structure of numbers up to 100
	› identify, read, and write whole numbers up to at least 10	› identify, read, and write whole numbers up to at least 20	› identify, read, and write whole numbers up to at least 100
	› compare and order whole numbers up to at least 10 and ordinal numbers (1st, 2nd, 3rd), using words	› compare and order whole numbers up to at least 20 and ordinal numbers (1st, 2nd, 3rd), using words or numerals with suffixes	› compare and order whole numbers up to at least 100
	› partition up to 5 objects, and then up to 10 objects, using a systematic approach and noticing patterns in the sequence	› partition and regroup up to 20 objects in different ways, using a systematic approach and noticing patterns	› partition and regroup whole numbers up to at least 100, using a systematic approach and noticing patterns

Year 3 *Informed by formative assessment, teach students to:*	**Teaching methods**
	Use the following when representing number structure:
› estimate to the nearest 10 the number of objects in a collection of less than 100	› dot patterns, 10s frames, fingers (years 0–2) › materials that can be grouped in 10 (e.g., iceblock sticks) (year 3)
› count to 1,000, forwards and backwards in 1s, 2s, 3s, 5s, 10s, and 100s, from any number	› number lines, 100s boards, number flip charts, 1,000s books
› recognise and represent the base ten structure of numbers up to 1,000	› 10s frames › ice block sticks, place-value (PV) blocks, PV money, PV houses, arrow cards
› identify, read, and write whole numbers up to at least 1,000	› 100s boards, PV houses, number fans, number cards, words and numerals
› compare and order whole numbers up to at least 1,000	› 100s boards (years 0–2) › number flip chart, PV houses, number cards, digit cards, number lines
› partition and regroup whole numbers up to at least 1,000, using a systematic approach and noticing patterns	› multilink cubes to break into groups, bead strings, 10s frames (years 0–1) › an abacus (year 2) › PV blocks, PV money (year 3) › partitioning diagrams, tables for recording patterns

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Number structure	› use the mathematical processes to: – generalise patterns and structures to quantify groups without counting – investigate different ways numbers can be partitioned – explain and justify using vocabulary that identifies quantities when ordering and comparing numbers and patterns (e.g., more than, same as, less than, between) – connect ordinal numbers with the counting sequence (year 1)		› use the mathematical processes to: – connect with algebra number patterns and te reo Māori or other languages with an explicit base 10 number structure – generalise the PV structure to compare and order numbers – investigate different ways numbers can be partitioned and recorded – explain and justify the structure of numbers using PV language
Operations		› use estimation to predict and to check the reasonableness of calculations	› use estimation to predict and to check the reasonableness of calculations
			› identify the nearest tens to any whole number up to 100
	› join and separate groups of up to a total of 10 objects, and find the result by grouping and counting	› join and separate groups of up to a total of 20 objects, and find the difference between groups by grouping and counting (e.g., 9 + 6; 7 + __ = 11)	› add and subtract numbers up to 100 without renaming (e.g., 53 + 21; 55 – 32)
		› multiply and divide by making equal groups and using grouping or counting	› multiply and divide by grouping and skip counting

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› use the mathematical processes to: – connect with algebra number patterns and te reo Māori or other languages with an explicit base 10 number structure – generalise the PV structure to compare and order numbers – investigate different ways numbers can be partitioned and recorded – explain and justify the structure of numbers using PV language	› discrete materials, PV materials › digit cards, patterns › number lines, words, equations.
	Use the following when representing operations:
› use estimation to predict and to check the reasonableness of calculations	› language for support (e.g., more or less than, close to)
› round whole numbers up to 1,000 to the nearest hundreds and tens	› number lines marked with the multiples of 10 or 100, progressing to unmarked number lines › 100s boards (year 2)
› add and subtract 2- and 3-digit numbers without renaming and without a change-unknown (e.g., 148 – 23; 235 + 121)	› discrete materials, number lines (years 0–2) › horizontal and vertical methods (year 3)
› multiply a 1- or 2-digit number by a 1-digit number, without renaming (e.g., 4 × 6; 2 × 23)	› pictures and diagrams (years 1–3) › discrete materials and number lines with grouping (years 1–2) › arrays, PV materials (year 3)
› divide whole numbers by a 1-digit divisor with no remainders, by grouping and using the inverse relationship with multiplication (e.g., 32 ÷ 4)	› pictures, diagrams, number lines › bar models, multilink cubes

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Operations	› use the mathematical processes to: – connect and use addition and subtraction in a range of situations – generalise the key ideas of counting when quantifying and when finding the total, difference, fair share, and comparisons – investigate word problems and the language used to describe an operation – explain and justify ways of quantifying, including counting, subitising, groupings, and sharing		› use the mathematical processes to: – connect and use addition, subtraction, multiplication, and division in a range of situations – generalise the use of the commutative property when solving addition problems – investigate word problems and identify an operation to use – explain and justify ways of quantifying, including estimation, groupings, and known efficient methods
Rational numbers		› identify and represent halves and quarters as fractions of sets and regions, using equal parts of the whole	› identify, read, write (using symbols and words), and represent halves, thirds and quarters as fractions of sets and regions, using equal parts of the whole
			› directly compare two fractions involving halves, thirds, and quarters
		› find a half or quarter of a set using equal sharing and grouping	› find a half, quarter, or third of a set by identifying groups and patterns (rather than sharing by ones)
			› identify, from part of a set or shape, the whole set or shape

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› use the mathematical processes to: – connect and use addition, subtraction, multiplication, and division in a range of situations – generalise the use of the commutative property when solving addition problems – investigate word problems and identify an operation to use – explain and justify ways of quantifying, including estimation, groupings, and known efficient methods	› materials, pictures › number lines, arrays › word problems, equations.
	Use the following when representing rational numbers:
› identify, read, write, and represent halves, thirds, quarters, fifths, sixths, and eighths as fractions of sets and regions, using equal parts of the whole	› a range of continuous materials (bar models) and discrete materials (sets of objects) › words, fraction symbols
› compare and order fractions involving halves, quarters, and eighths and identify when two fractions are equivalent	› a range of continuous materials (bar models, fraction tiles) and discrete materials (sets of objects)
› find a unit fraction of a whole (e.g., $\frac{1}{3}$ of 15)	› discrete materials › bar models (including paper strips) to show a whole and fractions to show partitions (year 3)
› identify, from a unit fraction part of a set or amount, the whole set or amount	› discrete materials › bar models (year 3)
› add unit fractions with the same denominator (e.g., $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$)	› fraction tiles, paper fraction strips, equations

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Rational numbers		› use the mathematical processes to: – connect fractions in measurement and geometry situations – investigate practical situations involving sharing, partitioning and identifying fractions – explain and justify ways to equal share.	› use the mathematical processes to: – connect a unit fraction of a quantity to division by a denominator – investigate different ways fractions can be represented and partitioned – explain that in a fraction the denominator indicates the number of parts a whole has been divided into, and the numerator the number of fractional parts
Financial maths			› recognise and order NZ denominations up to $20 according to their value, make groups of 'like' denominations, and calculate their value
			› use the mathematical processes to: – connect to place value – investigate appropriate financial situations.

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› use the mathematical processes to: – connect a unit fraction of a quantity to division by a denominator – investigate different ways fractions can be represented and partitioned – explain that in a fraction the denominator indicates the number of parts a whole has been divided into, and the numerator the number of fractional parts	› discrete and continuous materials › words, pictures, symbols, number lines.
	Use the following when representing financial maths:
› make amounts of money using one- and two-dollar coins and 5-, 10-, 20-, 50-, and 100-dollar notes	› play money (coins and notes)
› use the mathematical processes to: – connect to addition and subtraction when calculating amounts – investigate appropriate financial situations.	› play money (coins and notes).

Algebra

	6 months *Informed by formative assessment, teach students to:*	**Year 1** *Informed by formative assessment, teach students to:*	**Year 2** *Informed by formative assessment, teach students to:*
Generalising number properties		› identify addition facts up to 10 and their corresponding subtraction facts (families of facts), including doubles and halves	› recall addition facts up to 10, and identify addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves
		› explore adding 0 to or subtracting 0 from a number	› explore multiplying a number by 0 and 1 and dividing a number by 1
		› explore the commutative property of addition (e.g., 5 + 4 = 4 + 5)	› identify the commutative property of addition (e.g., 5 + 4 = 4 + 5)
			› identify the relationship between skip counting and multiplication facts for 2s, 5s, and 10s
		› use the mathematical processes to investigate the relationship between addition and subtraction	› use the mathematical processes to: – generalise subtraction problems beyond recalled facts by looking for patterns – investigate patterns using choral counting, materials, the recording of multiples, and the relationships between skip counting and multiplication and division facts

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing number properties:
› recall addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves	› materials, including 10s frames and multilink cubes › games
› explore dividing a number by itself, and why we cannot divide by 0 (e.g., by trying to solve 0 × __ = 5)	› word problems › materials
› use the additive identity (e.g., 4 + 0 = 4 and 5 – 0 = 5), multiplicative identity (e.g., 5 × 1 = 5 and 4 ÷ 1 = 4), and commutative property	› materials, including 10s frames and blocks › the concept of equality (years 1–2) › word problems and materials (year 3)
› recall multiplication and corresponding division facts for 2s, 3s, 5s, and 10s	› 100's boards patterns, choral counting, games and number lines (year 2) › games, families of facts, table grids (year 3)
› use the mathematical processes to: – generalise subtraction problems beyond recalled facts by looking for patterns – investigate patterns using choral counting, materials, the recording of multiples, and the relationships between skip counting and multiplication and division facts	› materials, pictures, word problems, equations, families of facts (to show addition and subtraction, multiplication and division).

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Equations and relationships		› solve true or false number sentences and open number sentences involving addition and subtraction of 1-digit numbers, using an understanding of the equal sign (e.g., 9 – 6 = 8 – __ ; 7 – 5 = 6 – 4 (T or F?)	› solve true or false number sentences and open number sentences involving addition and subtraction of 1- and 2-digit numbers, using an understanding of the equal sign (e.g., 18 + __ = 17 + 6; 17 = 25 (T or F?)
	› copy, continue, create, and describe a repeating pattern with two elements	› copy, continue, create, and describe a repeating pattern with three elements, and identify missing elements in a pattern	› recognise and describe the unit of repeat in a repeating pattern, and use it to predict further elements using the ordinal position
	› use the mathematical processes to: – generalise when noticing that repeated patterns constructed in different ways are the same pattern (e.g., 'red, blue, red, blue' and 'hop, jump, hop, jump' are both ABAB patterns) – investigate repeating patterns in a range of contexts – explain and justify how a pattern is repeating		› use the mathematical processes to: – generalise using the unit of repeat and ordinal position to identify further elements in a pattern – investigate repeating patterns in a range of contexts – explain and justify how a pattern is repeating or growing, and predict future terms in the pattern
Algorithmic thinking		› sort objects into two groups, following a simple rule	› follow a set of instructions to sort numbers or objects according to a simple rule
			› give step-by-step instructions, and identify and correct errors as they are followed
		› use the mathematical processes to investigate appropriate situations.	

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing equations and relationships:
› solve true or false number sentences and open number sentences involving addition and subtraction, using an understanding of the equal sign	› balance scales and discrete materials (year 1) › 10s frames and discrete materials (year 2) › word problems with comparisons (year 3)
› recognise, continue, and create growing patterns, and describe a rule to explain a pattern	› a range of materials with attributes (e.g., size, colour, texture, shape, movement, sound) › tables (years 2–3) › discrete objects, counters, blocks (year 3)
› use the mathematical processes to: – generalise using the unit of repeat and ordinal position to identify further elements in a pattern – investigate repeating patterns in a range of contexts – explain and justify how a pattern is repeating or growing, and predict future terms in the pattern	› materials, words, symbols, movements, drawings, tables.
	Use the following when representing algorithmic thinking:
› follow, and create patterns from, rules or simple algorithms	› discrete objects (year 1) › drawing and describing a representation of instructions › direction cards to form a sequence › flow diagrams (years 2–3)
› formulate a familiar routine or basic task as a set of precise, step-by-step instructions (i.e., an algorithm)	› drawing and describing a representation of instructions › direction cards to form a sequence, sequence steps, flow diagrams
› use the mathematical processes to investigate appropriate situations.	› verbal instructions, symbols, drawings › direction cards.

Measurement

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Measuring	› directly compare two objects by an attribute (e.g., length, mass (weight), capacity)	› compare the length, mass (weight), temperature, volume, and capacity of objects directly and indirectly (e.g., by comparing each of them with another object and using the object repeatedly)	› estimate and use an informal unit repeatedly to measure the length, mass (weight), volume, or capacity of an object
			› compare and order several objects using informal units of length, mass (weight), volume, or capacity
			› turn, and describe how far an object or person has turned, using half and quarter turns as benchmarks
	› use the mathematical processes to: – investigate ways to directly and indirectly compare – explain and justify, using the language of comparison (more, less, longer, shorter, heavier, lighter)		› use the mathematical processes to: – connect to ordering and comparing numbers – investigate a range of practical measurement situations, including ways of measuring by different cultures – explain and justify, using the same informal unit when measuring

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing measuring:
› estimate and then reliably measure length, capacity, and mass (weight), using metric units (e.g., from tools with labelled markings)	› physical comparisons (years 0–1) › balance scales, a range of capacity containers (years 1–2) › rulers, measuring jugs and containers, scales (year 3)
› compare and order objects using metric units of length, mass (weight), or capacity	› identical units (e.g., blocks, hands, paper clips, cups (year 2) › metric units on appropriate tools (year 3)
› turn, and describe how far an object or person has turned, using half, quarter, and three-quarter turns as benchmarks	› physical objects › themselves
› use the mathematical processes to: – connect to base 10 place value, ordering and comparing numbers – investigate a range of practical measurement situations, including ways of measuring by different cultures – explain and justify, using the labelled markings on tools	› physical objects, progressing from informal units to appropriate metric tools.

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Perimeter, area, and volume			› visualise, estimate, and measure the perimeter and area of 2D shapes, using informal units
			› use the mathematical processes to: – connect with groupings, addition, and known multiplication facts – investigate practical familiar contexts – explain and justify the importance of using the same unit when measuring
Time	› connect days of the week to familiar events and daily routines (e.g., the class timetable)	› identify how the passing of time is measured in years, months, weeks, days, hours, minutes, and seconds › name and order the days of the week, and sequence events in a day using everyday language of time	› name and order the months and seasons › describe duration using months, weeks, days, and hours › use a calendar to identify the date and to determine the number of days in each month
		› tell the time to the hour using the language of 'o'clock'	› tell the time to the hour and half-hour, using the language of 'past' and 'o'clock'
	› use the mathematical processes to: – connect daily routines and familiar events to days of the week and months of the year – investigate a calendar (its days, weeks, and months) and how long it takes to do tasks (i.e., duration).		› use the mathematical processes to: – connect half past, quarter to, and quarter past to fractions; and daily routines and familiar events to days of the week and months of the year – investigate calendars (their days, weeks, and months).

Year 3 *Informed by formative assessment, teach students to:*	**Teaching methods**
	Use the following when representing perimeter, area, and volume:
› visualise, estimate, and measure: – the perimeter of polygons using metric units – the area of 2D shapes covered with squares of identical size – the volume of rectangular prisms (cuboids) by filling them with identical units	› 2D shapes as a unit of measure for perimeter (e.g., ice block sticks) (year 2) › informal units for measuring area (e.g., blocks, squares, sticky notes) › square grids and rulers (year 3)
› use the mathematical processes to: – connect with groupings, addition, and known multiplication facts – investigate practical familiar contexts – explain and justify the importance of using the same unit when measuring	› materials, square grids, blocks, rulers.
	Use the following when representing time:
› use a calendar to work out the number of days, weeks, or months until important events	› pictorial timetables (years 0–1) › cards for ordering days › calendars › the classroom daily timetable
› tell the time to the hour, half hour, and quarter past and quarter to the hour	› analogue and digital clocks
› use the mathematical processes to: – connect half past, quarter to, and quarter past to fractions; and daily routines and familiar events to days of the week and months of the year – investigate calendars (their days, weeks, and months).	› timetables › analogue and digital clocks › a calendar.

Geometry

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Shapes	› identify, sort by one feature, and describe familiar 2D shapes	› identify, describe, and classify familiar 2D and 3D shapes presented in different orientations, including triangles, circles, rectangles (including squares), cubes, cylinders, and spheres	› identify, describe, and classify the properties of 2D and 3D shapes including ovals, semicircles, polygons (e.g., hexagons, pentagons), rectangular prisms (cuboids), pyramids, hemispheres, and cones, using the properties of shapes
	› use the mathematical processes to: – connect 2D and 3D shapes in the environment – investigate ways of sorting 2D and 3D shapes into groups – explain, justify, and compare how shapes have been grouped		› use the mathematical processes to: – connect right angles to square corners in shapes and objects – investigate properties of 2D and 3D shapes, including lines of symmetry – explain and justify the classification of shapes into groups based on their properties
Spatial reasoning	› compose by trial and error an outlined target shape using smaller shapes, and decompose a shape into smaller shapes	› anticipate which smaller shapes might be used to compose a target shape, and then check by making the shape	› anticipate which smaller shapes might be used to compose and decompose a target shape, and then check by making the shape
		› slide, flip, and turn 2D shapes to make a pattern	› recognise lines of symmetry in patterns or pictures, and create or complete symmetrical pictures or patterns

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing shapes:
› visualise, identify, compare, and classify 2D and 3D shapes using the properties of shapes including lines of symmetry	› a range of 2D and 3D shapes › tactile materials › digital tools
› identify right angles in shapes and objects	› 2D and 3D shapes › objects in the environment
› the mathematical processes to: – connect right angles to square corners in shapes and objects – investigate properties of 2D and 3D shapes, including lines of symmetry – explain and justify the classification of shapes into groups based on their properties	› materials › digital tools › vocabulary.
	Use the following when representing spatial reasoning:
› compose and decompose 2D shapes using the properties of shapes (e.g., lines of symmetry), other shapes, side lengths, and angles	› pattern blocks, attribute shapes, paper shapes, playdough, tangrams › discrete target shapes (year 0) › continuous target shapes (years 1–3)
› predict the result of a one-step transformation on 2D shapes	› 2D shapes, paper folding › mirrors › symmetrical object outlines (discrete and continuous) › painting, art-related tasks › predicting the result of a transformation

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Spatial reasoning	› use the mathematical processes to: – investigate how shapes can be flipped and turned to make patterns – explain and justify how new shapes can be created, using the names and properties of the shapes and spatial vocabulary		› use the mathematical processes to: – connect quarter, half, and three-quarter turns to fractions – generalise about 2D shapes (e.g., how they can be partitioned into smaller shapes, and how, when orientated in different directions (flip, turn), their properties do not change) – investigate transformation (flip, slide, turn) and lines of symmetry in pictures, patterns, and the environment – explain and justify how shapes have been used to create new shapes
Pathways	› follow instructions to move to a familiar location or locate an object	› follow and give instructions to move to a familiar location or locate an object	› follow and give instructions to move people or objects to a different location, using direction, distances (e.g., number of steps), and half and quarter turns
		› use pictures, diagrams, or stories to describe the positions of objects and places	› interpret diagrams to describe the positions of objects and places in relation to other objects and places
	› use the mathematical processes to: – investigate ways of moving to different locations by following verbal instructions and simple diagrams and maps.	› use the mathematical processes to: – connect half and quarter turns with fractions – investigate ways of moving to different locations by following verbal instructions and simple diagrams and maps.	

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› use the mathematical processes to: – connect quarter, half, and three-quarter turns to fractions – generalise about 2D shapes (e.g., how they can be partitioned into smaller shapes, and how, when orientated in different directions (flip, turn), their properties do not change) – investigate transformation (flip, slide, turn) and lines of symmetry in pictures, patterns, and the environment – explain and justify how shapes have been used to create new shapes	› 2D shapes.
	Use the following when representing pathways:
› follow and create a sequence of step-by-step instructions (an algorithm) for moving people or objects to a different location	› familiar locations › speaking frames for simple instructions (e.g., sentence starters) › spatial language (to support following and giving instructions) › directions, distance, turns (years 2–3)
› interpret, draw, and use simple maps to locate objects and places relative to other objects and places	› story books, familiar locations › spatial language (to support following and giving instructions) › simple maps › modelling how to draw physical representation of a simple map (year 3)
› use the mathematical processes to: – connect quarter, half, and three-quarter turns to fractions – investigate ways of moving to different locations by following verbal instructions and simple diagrams and maps.	› spatial vocabulary › symbols › simple maps.

Statistics

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Problem		› pose summary investigative questions that classify objects or individuals into groups or categories (e.g., colour, brand), and anticipate what the data might show	› pose summary investigative questions about a group for which the data will have categorical variables, and anticipate what the data might show (e.g., which outcomes might be more frequent than others)
		› use the statistical processes to: – pose summary investigative questions about a group and for which the data will have categorical variables – investigate an area of interest and things students are curious about	› use the statistical processes to: – pose an investigative question with support – investigate an area of interest and things students are curious about
Plan		› collect data for one variable by making observations or questioning others, and discuss how the data-gathering process might affect other people	› use survey and data-collection questions to collect data, identify who and what the data measures, and discuss how the data-gathering process might affect other people
		› use the statistical processes to: – plan ways of collecting data and survey questions, with support – investigate different survey questions and how they can be interpreted by others	
Data		› collect categorical data for one variable	› collect categorical data for more than one variable
		› use the statistical processes to: – collect data using data cards, recording, and tally sheets – investigate different ways of collecting data	

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› pose summary investigative questions about everyday situations, using categorical data and discrete numerical (whole number) data, including about identifying the variable and the group of interest, and anticipate what the data might show	Support students to pose questions about an area of interest.
› use the statistical processes to: – pose an investigative question with support – investigate an area of interest and things students are curious about	
› use survey and data-collection questions to collect data, identify who and what the data measures, and discuss how the data-gathering process might affect other people	Demonstrate data collection methods.
› use the statistical processes to: – plan ways of collecting data and survey questions, with support – investigate different survey questions and how they can be interpreted by others	
› collect, record, and sort data, or use secondary data sources provided by someone else	Represent data using data cards, recording sheets, and tally sheets.
› use the statistical processes to: – collect data using data cards, recording, and tally sheets – investigate different ways of collecting data	

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Analysis		› create and describe data visualisations (e.g., picture graphs, physical dot plots) for categorical data, giving the frequency for each category	› create and describe data visualisations (e.g., picture graphs, dot plots) for categorical data, comparing the frequencies of categories
		› use the statistical processes to: – investigate how different representations (e.g., a picture graph and dot plot) show the same information – explain and justify what a graph shows using 'I notice …' statements	
Conclusion		› choose statements that best answer the investigative question	
		› use the statistical processes to: – connect descriptions with data visualisations and analysis questions with features of the visualisations – investigate ways of reflecting on findings to determine if they make sense with what they already know – explain why some statements answer the investigative question and some do not	
Statistical literacy		› agree or disagree with others' statements about simple data visualisations (e.g., pictographs, physical dot plots).	› match statements made by others with features in simple data visualisations, and agree or disagree with the statements.

Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› create and describe data visualisations (e.g., picture graphs, dot plots, bar graphs) for categorical and discrete numerical data	Demonstrate creating data visualisations and describing what a graph shows. Use data cards, picture graphs, frequency tables, and dot plots (years 1–2). Use frequency tables, pictographs, and bar graphs (year 3). Use 'I notice' statements.
› use the statistical processes to: – represent data using data cards, frequency tables, picture graphs, pictographs, dot plots, and bar graphs – investigate how different representations (e.g., a picture graph and dot plot) show the same information – explain and justify what a graph shows using 'I notice …' statements	
› choose statements that best answer the investigative question, reflect on findings, and compare them with anticipated outcomes	Demonstrate making statements about data visualisations. Show the structure of a comparative statement from data.
› use the statistical processes to: – connect descriptions with data visualisations and analysis questions with features of the visualisations – investigate ways of reflecting on findings to determine if they make sense with what they already know – explain why some statements answer the investigative question and some do not	
› identify relevant features in others' data visualisations, connect these to descriptive statements, agree or disagree with the statements, and suggest improvements	Demonstrate making claims about data visualisations, including giving reasons for agreeing or disagreeing with a claim.
› use the statistical processes to explain and justify, using agree-with and disagree-with descriptive statements, and suggest ways to improve.	

Probability

	6 months *Informed by formative assessment, teach students to:*	Year 1 *Informed by formative assessment, teach students to:*	Year 2 *Informed by formative assessment, teach students to:*
Probability investigations		› engage in stories or games that involve chance-based situations and: – decide if something will happen, won't happen, or might happen – identify possible and impossible outcomes (e.g., what might happen next)	› engage in chance-based investigations about games and everyday situations to: – identify possible outcomes – collect and record data – create visualisations for frequencies of outcomes (e.g., lists, picture, graphs) – describe what these data visualisations show – answer chance-based investigative questions – notice variations in outcomes (e.g., how often each of the numbers on a dice come up)
Critical thinking in probability			› agree or disagree with the statements made by others about chance situations
		› use the statistical processes to: – connect relative frequency in words (e.g., two out of three) to fractions (e.g., 2/3) – investigate games of chance and list possible outcomes – use the statistical enquiry cycle (PPDAC) for chance-based investigations – explain, justify, and use the language of probability (impossible, unlikely, possible, likely, certain) and its ordering from impossible to certain.	

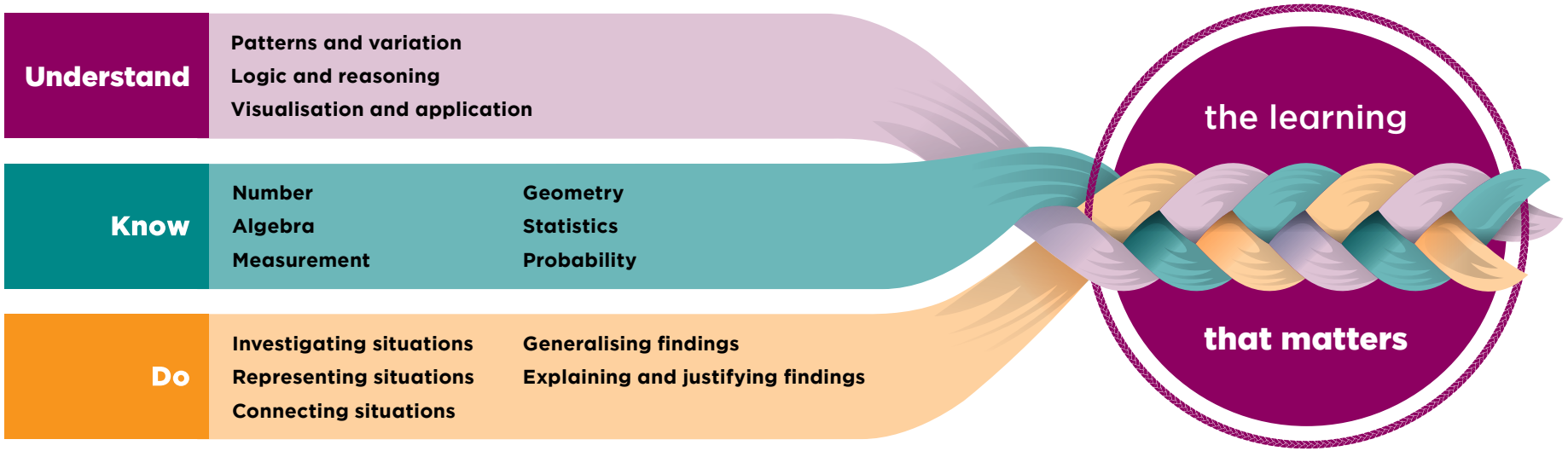
Year 3 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› engage in chance-based investigations about games and everyday situations to: – anticipate what might happen – identify possible outcomes – collect and record data – create data visualisations for frequencies of possible outcomes – describe what these visualisations show – answer investigative questions – reflect on anticipated outcomes – notice variations in outcomes	Play games of chance using physical objects (e.g., dice, coins, spinners, pulling things out of a hat). Create tables for frequencies, lists, and outcomes.
› explain and question statements about chance-based situations, with reference to data	Demonstrate making claims about data visualisations, including giving reasons for agreeing or disagreeing with a claim.
› use the statistical processes to: – connect relative frequency in words (e.g., two out of three) to fractions (e.g., 2/3) – investigate games of chance and list possible outcomes – use the statistical enquiry cycle (PPDAC) for chance-based investigations – explain, justify, and use the language of probability (impossible, unlikely, possible, likely, certain) and its ordering from impossible to certain.	Represent probability using materials, dice, coins, spinners, and items in a bag. Represent outcomes using drawings, tally charts, lists, and tables.

Phase 2
Years 4–6

Expanding horizons of knowledge and collaborating
Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi

Progress outcome by the end of year 6

In phase 2, students weave together Understand, Know, and Do and use a variety of visual representations to model number operations and solve word problems. They connect and extend their reasoning about whole numbers to fractions and decimals, and they visualise and classify angles, using benchmarks to justify their classifications. They also apply their understanding of number operations to reason about perimeter and area and apply comparative reasoning to investigate variations in shapes and data.



Understand
Big ideas

As they develop their maths capabilities and knowledge, students build on their understanding that the world is full of **patterns** and of relationships in which **variation** occurs. They use the structures of mathematics and statistics to support noticing, exploring, and describing different types of patterns and relationships, enabling insights, generalisations, and predictions. By engaging with maths concepts, students develop **logical** reasoning and critical thinking skills that enable them to evaluate information, question assumptions, and present reasoning with clarity.

Statistical **reasoning** from observation allows students to explore what is probable and to draw reasonable conclusions. They continue to develop the ability to **visualise** maths ideas in order to understand and interact with abstract concepts. They **apply** maths ideas to help understand familiar situations and to support decision making and the communication of ideas.

Know
Content and concepts

Number | Mātauranga tau

Students know that in our **number system** each place value is a power of 10, and this continues infinitely. To the right, the system continues beyond ones, to create decimals (tenths, hundredths, thousandths); the decimal point is placed between the ones column and the tenths column. **Estimation and rounding** support checking the reasonableness of solutions of operations involving whole numbers, fractions, and decimals.

Students know that to **calculate expressions** that have more than one operation, operations inside brackets (grouped together) are done first, then powers or exponents. If there are multiplication and division, these are done in left-to-right order; addition and subtraction are also done in left-to-right order. Students use the mnemonic GEMA: grouped, exponents, multiplication, addition. Division can be partitive (the number of shares is known) or quotative (the size of the shares is known).

Students also know that **fractions** can represent one number divided by another, operate on quantities, and be larger than 1. Improper fractions can also be written as a mixed number represented as a whole number and a fraction, combined with a hidden addition. In simplified fractions, the numerator and denominator have no common factors; if the denominator of a simplified fraction is 1, then it can be written as a whole number. **Decimals** are fractions that have powers of 10 as their denominators and that can be written as numbers using a decimal point. A **percentage** is a fraction with a denominator of 100.

Algebra | Taurangi

Students know the **properties** of operations: the commutative and associative properties only work for addition and multiplication (not for subtraction and division), the associative property applies to addition and multiplication, and the distributive property applies to multiplication over addition and subtraction. Students know that the equal (=) and inequality (<, >) signs show relationships and that applying the same operation to both sides of an equation preserves the balance of the equation.

Students know that in a **pattern**, the relationship between the **ordinal position** and the corresponding element can be used for finding the pattern rule. Any element can be found by knowing the position, and any position can be found from the element. Tables and XY graphs provide a way of organising the positions and elements of a pattern to reveal relationships or rules. An **algorithm** is an ordered list of instructions for solving a problem.

Measurement | Ine

Students know that, like our place-value number system, the **metric measurement system** is based on powers of 10 and that appropriate metric units are used to quantify length, area, volume, capacity, mass (weight), temperature, and duration. Measurements can include whole units and parts of units. Different **measurement tools and scales** use different-sized units, and the unit must be recorded with the amount. If a measurement starts at 0, the point on the scale where it ends tells us the measurement. Angles measure parts of a full turn and use the unit of degrees.

Geometry | Āhuahanga

Students know that two- and three-dimensional shapes have consistent **properties** that can be used to define, compare, classify, predict, and identify relationships between shapes. Shapes can be **transformed** by rotation, reflection, translation, and resizing (when they are enlarged or reduced). Lines of symmetry can

be horizontal, vertical, and diagonal. **Three-dimensional shapes** can be composed of connected two- or three-dimensional shapes. Students also know that **position** can be described using known environmental features and signs from the natural world. Maps can use grid references to specify the position of locations, scales to show distances, and connections to show pathways.

Statistics | Tauanga

Students know that data about people and the natural world must be collected, used, and stored carefully. The **statistical enquiry cycle** (PPDAC) can be used in **summary, comparison, and time-series investigations**. A comparison investigation compares similarities and differences for a variable across two or more groups, and a time-series investigation considers a variable that changes over time. **Numerical variables** can be counted or measured; discrete numerical variables are counted, continuous numerical variables are measured. A **prediction** or assertion involves thinking about what data will show before it is collected or analysed. Data is not always accurately recorded; it needs to be checked for errors and may need correcting. Alternative **data visualisations** for the same data can lead to different insights.

Probability | Tūponotanga

Students know that the statistical enquiry cycle (PPDAC) can be used for **chance-based investigations**. Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain), can be used to describe situations that involve **uncertainty**, and help make decisions. The **probability of an outcome** in a chance-based investigation is the number of times the outcome occurs divided by the total number of outcomes, where all possible outcomes can be listed (theoretical probability), or the relative frequency of the outcome from a probability experiment (estimated probability).

Do
Practices

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing a question, plan an investigation pathway, and follow it step by step. They can identify relevant prior knowledge, givens, and relationships to support the investigation. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

Students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisations to mentally represent and manipulate objects and ideas.

Connecting situations | Te tūhono pūāhua

Students can suggest connections between ideas, approaches, and different representations. They connect new ideas to things they already know. They make connections with ideas in other learning areas and with familiar cultural, linguistic, and historical contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

Students can recognise and explore patterns, and make conjectures and draw conclusions about them. They can identify relationships, including similarities, differences, and new connections. They look for patterns and regularities that might be applied in another situation or always be true. They make and test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

Students can make statements, give explanations inductively based on observations or data, and make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others, and they present reasoned explanations and arguments for an idea, solution, or process.

Phase 2
Years 4–6

Teaching sequence

Expanding horizons of knowledge, and collaborating
Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi

Throughout phase 2, encourage students to see themselves as capable, confident, and competent maths thinkers whose ideas are valued, who treat mistakes as part of the learning process, and whose capabilities in maths can develop over time with consistent effort. Confidence is built through experiencing success and developing competence and understanding. Over phase 2, students collaborate with others to expand their knowledge and understanding. Support this by working with the class to establish expectations and responsibilities when it comes to working together as peers, sharing thinking, and agreeing or disagreeing about maths learning.

Continuously monitor students' reasoning, questions, and use of representations, and act quickly to respond to any misconceptions. Ensure your teaching builds on what students already understand, know, and can do.

Throughout each day, provide opportunities for students to:

- **develop positive relationships with maths –** explore rich mathematical situations and maths tasks that are useful and meaningful to the class or community
- **actively listen to, reflect, and build on each other's thinking and learning –** use discourse-based tools and a range of open questions to facilitate productive discussions. Over the phase, encourage students to use evidence and examples to justify their claims and findings
- **experience explicit teaching of concepts and practices –** use worked examples and break down new learning into clearly explained manageable steps. Use and explain maths symbols and notation conventions, and how they work. Teach efficient written methods, including column methods for operations

- **select appropriate representations to share their working and reasoning –** over the phase, move students towards using pictures, diagrams, and mathematical notation such as equations and inequalities. Teach students which representations are most effective for visualising different types of information (e.g., number lines are important for visualising operations, differences, the comparative size of numbers, and rounding conventions)
- **actively learn maths vocabulary –** help students to connect the correct vocabulary to the learning purpose and problem. Ask them to use the correct vocabulary when explaining their findings and reasoning, and draw on their first and heritage languages
- **connect maths learning within and across contexts –** teach connected skills and concepts together (e.g., multiplication and division with area and volume). Make connections obvious and explicit by highlighting skills or concepts students have applied in other learning areas
- **visualise quantities, patterns, shapes, measurements, and space –** support students to visual by estimating the number of items in a group, using rounding or known benchmarks to make estimations, and noticing how a shape has been rotated, reflected, or is composed of other shapes
- **consolidate maths learning to develop fluency with time-sensitive statements in the teaching sequence –** use warm-up routines as a form of active recall that connects back to prior learning (e.g., a quick challenge, curly question, or game). Plan for students to develop fluency through practice, using a range of approaches
- **record learning in their maths book –** encourage students to record vocabulary, problem solving, summaries, goals, and reflections on their maths learning. This could include words, mathematical notation, and a range of representations.

The highlighted cells in the teaching sequence show critical steps that alert teachers to specific aspects of learning that are essential and time-sensitive. They support teachers to notice, recognise, and respond to student learning in a timely way.

Number

	Year 4 *Informed by formative assessment, teach students to:*	**Year 5** *Informed by formative assessment, teach students to:*
Number structure	› count to and within 1,000, from any multiple of 100, forwards and backwards in 25s and 50s	
	› recognise the base ten structure of numbers up to 10,000	› recognise the base ten structure of numbers up to 100,000
	› identify, read, write, compare, and order whole numbers up to 10,000	› identify, read, write, compare, and order whole numbers up to 100,000
		› identify factors of numbers up to 100
	› use the mathematical processes to: – connect with metric units that are powers of 10, and with decimal place value	› use the mathematical processes to: – connect with metric units that are powers of 10, and with decimal place value – investigate factors and multiples
Operations	› use rounding and estimation to predict and to check the reasonableness of calculations	› use rounding and estimation to predict and to check the reasonableness of calculations
	› round whole numbers to the nearest thousand, hundred, or ten, and round tenths to the nearest whole number	› round whole numbers to a specified power of 10, and round tenths and hundredths to the nearest whole number
	› add and subtract 2- and 3-digit numbers	› add and subtract whole numbers up to 10,000
	› multiply a 2-digit by 1-digit number and two 1-digit whole numbers (e.g., 5 × 46; 8 × 7)	› multiply a 3-digit by 1-digit number and two 2-digit whole numbers (e.g., 6 × 248; 37 × 84)

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing number structure:
	› patterns in a 100s board and 1,000s book › choral counting, captured and recorded on the board
› recognise the base ten structure of numbers up to 1,000,000	› place-value (PV) houses and materials
› identify, read, write, compare, and order whole numbers up to 1,000,000	› marked number lines, PV houses, materials
› identify square numbers and factors of numbers up to 125	› arrays, lists of factor pairs, multiplication charts
› use the mathematical processes to: – connect with metric units that are powers of 10, and with decimal place value – investigate factors and square numbers	› verbal and written numbers › PV materials, number lines.
	Use the following when representing operations:
› use rounding and estimation to predict and to check the reasonableness of calculations	› known facts › benchmarks
› round whole numbers to a specified multiple of powers of 10, and round tenths and hundredths to the nearest whole number or one decimal place	› number lines
› add and subtract any whole numbers	› materials › horizontal and vertical methods
› multiply multi-digit whole numbers (e.g., 54 × 112)	› jumps on a number line (year 4) › area-model and vertical methods

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Operations	› divide whole numbers by a 1-digit divisor, with no remainders (e.g., 65 ÷ 5)	› divide whole numbers by a 1-digit divisor, with a remainder (e.g., 83 ÷ 5 = 16, remainder 3)
	› use the mathematical processes to: – connect multiplication, division, and factors with area, volume, and perimeter – connect multiplication and division with proportional reasoning – generalise the use of inverse operations and the commutative and distributive properties, to check findings – investigate comprehending and solving word problems, deciding which operation to use and why – explain and justify findings, by connecting to estimates and other checking methods	
Rational numbers	› identify, read, write, and represent tenths as fractions and decimals	› identify, read, write, and represent tenths and hundredths as fractions and decimals
	› compare and order tenths as fractions and decimals, and convert decimals to fractions	› compare and order tenths and hundredths as fractions and decimals, and convert decimals to fractions
	› divide whole numbers by 10 to make decimals	› divide whole numbers by 10 and 100 to make decimals and whole numbers
	› for fractions with related denominators of 2, 4, and 8, 3 and 6, or 5 and 10: – compare and order the fractions – identify when two fractions are equivalent – represent the fractions in their simplest form	› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: – compare and order the fractions – identify when two fractions are equivalent – represent the fractions in their simplest form
	› convert, using number lines, between improper fractions and mixed numbers for fractions with denominators of 2, 3, 4, 5, 6, and 10	› convert between improper fractions and mixed numbers for fractions with denominators up to 10

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› divide whole numbers by a 1-digit divisor, with a remainder (e.g., 198 ÷ 7; 4154 ÷ 8)	› diagrams, known facts (year 4) › jumps on a number line › vertical method
› use the order of operations rule GEMA with grouping, addition, subtraction, multiplication, and division	› step-by-step layouts with only one equal sign per line
› use the mathematical processes to: – connect multiplication, division, and factors with area, volume, and perimeter – connect multiplication and division with proportional reasoning – generalise the use of inverse operations and the commutative and distributive properties, to check findings – investigate comprehending and solving word problems, deciding which operation to use and why – explain and justify findings, by connecting to estimates and other checking methods	› diagrams › number lines › area models › vertical and horizontal methods.
	Use the following when representing rational numbers:
› identify, read, write, and represent fractions, decimals (to two places), and percentages	› equivalence materials, number lines, decimal PV houses
› compare and order fractions, decimals (to two places), and percentages and convert decimals, and percentages to fractions	› double number lines › blank 100s boards
› multiply and divide numbers by 10 and 100 to make decimals and whole numbers	› PV houses
› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: – compare and order the fractions – identify when two fractions are equivalent – represent the fractions in their simplest form	› fraction walls (equivalence materials) and benchmarks, to show the relationship between two denominators for simplifying › number lines, for comparing and ordering
› convert between improper fractions and mixed numbers	› fraction walls and tiles › number lines

	Year 4 *Informed by formative assessment, teach students to:*	**Year 5** *Informed by formative assessment, teach students to:*
Rational numbers	› find a unit fraction of a whole number, using multiplication or division facts and where the answer is a whole number (e.g., $\frac{1}{5}$ of 40)	› find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., $\frac{2}{3}$ of 24)
	› identify, from a unit fraction part of a set, the whole set	› identify, from a fractional part of a set, the whole set
	› add and subtract fractions with the same denominators to make up to one whole or less than one whole (e.g., $\frac{3}{8} + \frac{3}{8} + \frac{2}{8} = 1$)	› add and subtract fractions with the same denominators, including to make more than one whole
	› add and subtract decimals to one decimal place	› add and subtract decimals to two decimal places
	› use doubling or halving to scale a quantity (e.g., to double or half a recipe)	› use known multiplication facts to scale a quantity
	› use the mathematical processes to: – connect equivalent fractions and decimals – connect decimal place value and operations with whole number place value and operations – connect decimals with measurement – investigate appropriate situations – explain and justify equivalent fractions – convert between mixed numbers and improper fractions – generalise that multiplying or dividing a number by a power of ten changes the position of the digits on a PV chart (years 5–6)	

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› find a fraction or percentage of a whole number where the answer is a whole number (e.g., $\frac{3}{8}$ of 48; 30% of $150)	› bar models › benchmarks (years 5–6)
› identify, from a fractional part of a set, the whole set	› discrete materials (year 4) › bar models
› add and subtract fractions with the same or related denominators (e.g., $\frac{1}{4} + \frac{1}{8}$)	› fraction tiles, number lines (years 4–5) › fraction tiles and number lines, to demonstrate renaming fractions with related denominators (year 6)
› add and subtract whole numbers and decimals to two decimal places	› horizontal methods (including number lines) and vertical methods
› use known multiplication and division facts to scale a quantity	› diagrams, known facts, horizontal and vertical methods
› use the mathematical processes to: – connect equivalent fractions and decimals – connect decimal place value and operations with whole number place value and operations – connect decimals with measurement – investigate appropriate situations – explain and justify equivalent fractions – convert between mixed numbers and improper fractions – generalise that multiplying or dividing a number by a power of ten changes the position of the digits on a place value chart (years 5–6)	› discrete and continuous materials.

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Financial maths	› make amounts of money using dollars and cents (e.g., to make 3 dollars and 70 cents)	› represent money values in multiple ways using notes and coins
	› estimate and calculate the total cost and change for items costing whole dollar amounts	› estimate the cost to the nearest dollar of items costing dollars and cents, and the change from the nearest ten dollars
	› use the mathematical processes to: – connect to rounding and addition and subtraction – investigate making amounts of money, using different denominations – investigate financial plans and decisions.	› use the mathematical processes to: – connect to rounding and addition and subtraction of decimals to two places – investigate making amounts of money, using different denominations – investigate financial plans and decisions.

Algebra

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Generalising number properties	› use inverse operations to solve multiplication and division problems	› use inverse operations to solve multiplication and division problems
	› explore the associative property with addition and multiplication	› explore why the commutative and associative properties do not work for subtraction and division
	› recall multiplication and corresponding division facts for 4s, 6s, 9s, and 10s	› recall multiplication facts to 10 × 10 and corresponding division facts

Year 6 *Informed by formative assessment, teach students to:*	**Teaching methods**
	Use the following when representing financial maths:
› solve problems involving purchases (e.g., ensuring they have enough money) › create simple financial plans (e.g., shopping lists, a family budget)	› play money › spreadsheets › tables
› calculate 10%, 25%, and 50% of whole dollar amounts (e.g., 50% of $280)	› known facts and rounding › bar models and benchmarks, to find percentages (year 6) › division, then subtracting from the whole (year 6)
› use the mathematical processes to: – connect to rounding, addition and subtraction of decimals to two places, and calculating a percentage of a whole amount – investigate making amounts of money, using different denominations – investigate financial plans and decisions.	› play money › written and mental methods.

Year 6 *Informed by formative assessment, teach students to:*	**Teaching methods**
	Use the following when representing number properties:
› use inverse operations to solve multiplication and division problems	› families of facts › bar models › equations, demonstrating working backwards
› use commutative, associative, and identity properties, deciding which operations they work for and which they don't	› array and area models for the commutative and associative properties
› recall multiplication facts to at least 10 × 10 and corresponding division facts	› families of facts, multiplication grids, arrays, number lines

	Year 4 *Informed by formative assessment, teach students to:*	**Year 5** *Informed by formative assessment, teach students to:*
Generalising number properties	› explore the distributive property of multiplication over addition (e.g., 7 × 8 = 7 × (5 + 3) = (7 × 5) + (7 × 3)	› explore the distributive property of multiplication over addition and subtraction (e.g., 6 × 18 = 6 × (20 – 2) = (6 × 20) – (6 × 2)
	› use the mathematical processes to: – generalise multiplication problems beyond recalled facts, by looking for patterns – investigate patterns in the multiples of times tables	
Equations and relationships	› form and solve true or false number sentences and open number sentences involving multiplication and division, using understanding of the equal sign (e.g., 5 × __ = 20; __ ÷ 3 = 6)	› form and solve true or false number sentences and open number sentences involving all four operations (e.g., 674 + 56 – k = 671)
	› recognise and describe the rule for a growing pattern using words, tables, and diagrams, and predict further elements in the pattern	› use tables to recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule in words, and predict further elements in the pattern
	› use the mathematical processes to: – investigate inverse operations to find missing numbers in equations and growing patterns (e.g., tivaevae) – explain and justify the relationship between the ordinal position and its corresponding element to find a pattern's rule.	
Algorithmic thinking		

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› use the distributive, commutative, and associative properties	› arrays, area models
› use the mathematical processes to: – generalise multiplication problems beyond recalled facts, by looking for patterns – investigate patterns in the multiples of times tables	› arrays,area models › families of facts.
	Use the following when representing equations and relationships:
› form and solve true or false number sentences and open number sentences involving all four operations, using equality or inequality (e.g., 8 × 7 < 8 × 5 + 8 (T or F?)	› things that balance › known facts › inverse operations › equal and inequality symbols
› use tables, XY graphs, and diagrams to recognise relationships in a linear pattern, develop a rule in words that identifies the constant amount of change between consecutive elements or terms in the pattern, and predict further elements in the pattern	› visually growing patterns, recording the position and term for each element in the sequence
› use the mathematical processes to: – investigate inverse operations to find missing numbers in equations and growing patterns (e.g., tivaevae) – explain and justify the relationship between the ordinal position and its corresponding element to find a pattern's rule	› diagrams, words › tables, graphs.
	Use the following when representing algebraic thinking:
› create and use algorithms for making decisions that involve clear choices (e.g., formulating a familiar routine as a set of step-by-step instructions)	› flowcharts › numbered instructions
› use the mathematical processes to: – connect to algorithms for operations – investigate situations that involve making decisions.	› step-by-step instructions › diagrams, flow charts.

Measurement

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Measuring	› develop personal benchmarks for estimation and measure length, area, mass (weight), capacity, and duration, using appropriate metric units	› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric units or a combination of units
	› use appropriate units to describe length, mass (weight), capacity, and time	› use the appropriate unit and tool for the task and the attribute being measured
	› use the metric measurement system to explore relationships between units	› use the metric measurement system based on powers of ten to explore relationships between units, including benchmark fractions and decimals
	› recognise that angles can be measured in degrees, using 90, 180, and 360 degrees as benchmarks	› describe an angle using the terms acute, right, obtuse, straight, and reflex, by comparing the angle with benchmarks of 90, 180, and 360 degrees
	› use the mathematical processes to: – connect measuring with place value and decimals, angles with fractions of a circle and degrees of turn, and benchmark fractions with measurements (e.g., 500ml = $\frac{1}{2}$ L) – investigate, using practical measuring situations (e.g., using scaled measurement instruments, reading angles using geometric software and protractors) – explain and justify the use of appropriate metric units for a given situation	

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing measuring:
› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric units or a combination of units	› rulers, scales, timers, thermometers, measuring jugs (always ensuring accurate use)
› select and use the appropriate unit and tool for the task and the attribute being measured	› units that match the situation
› convert between common metric units for length, mass (weight), and capacity; and use decimals to express parts of wholes in measurements	› PV and decimal PV houses
› visualise, measure, and draw (to the nearest degree), the amount of turn in angles up to 360 degrees	› protractors, to demonstrate measuring and drawing (year 6)
› use the mathematical processes to: – connect measuring with place value and decimals, angles with fractions of a circle and degrees of turn, and benchmark fractions with measurements (e.g., 500ml = $\frac{1}{2}$ L) – investigate, using practical measuring situations (e.g., using scaled measurement instruments, reading angles using geometric software and protractors) – explain and justify the use of appropriate metric units for a given situation	› measurement tools › PV materials.

	Year 4 *Informed by formative assessment, teach students to:*	**Year 5** *Informed by formative assessment, teach students to:*
Perimeter, area, and volume	› visualise, estimate, and calculate: – the perimeter of polygons using metric units – the area of shapes covered with squares or half squares – the volume of shapes filled with centicubes, taking note of layers and stacking	› visualise, estimate, and calculate: – the perimeter of polygons – the area of shapes covered with squares or partial squares – the volume of rectangular prisms, taking note of layers and stacking
	› use the mathematical processes to: – connect area with multiplication arrays and the commutative property of multiplication – generalise the formula for finding the area and volume of rectangles and rectangular prisms – investigate practical contexts for finding perimeter, area, and volume	
Time		› describe the differences in duration between units of time (e.g., days and weeks, months and years)
	› tell the time to the nearest 5 minutes, using the language of minutes past the hour and to the hour	› solve duration-of-time problems involving 'am' and 'pm' notation
	› use the mathematical processes to: – connect units of time to fractions – investigate calendars, timetables, and schedules to work out the duration between events, or the start and end times for events.	

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing perimeter, area, and volume
› visualise, estimate, and calculate the area of rectangles and right-angled triangles and the volume of rectangular prisms, by applying multiplication	› square grids › rulers › 2D shapes
› use the mathematical processes to: – connect area with multiplication arrays and the commutative property of multiplication – generalise the formula for finding the area and volume of rectangles and rectangular prisms – investigate practical contexts for finding perimeter, area, and volume	› written methods with clearly laid out working.
	Use the following when representing time:
	› word problems, for converting weeks and months into days and vice versa
› convert between units of time and solve duration-of-time problems, in both 12- and 24-hour time systems	› digital and analogue clocks (year 4) › subtracting time to calculate duration › inclusive counting (e.g., for the number of days between now and next Tuesday, start counting from today) (years 5–6)
› use the mathematical processes to: – connect units of time to fractions – investigate calendars, timetables, and schedules to work out the duration between events, or the start and end times for events.	› digital and analogue clocks.

Geometry

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Shapes	› identify, classify, and describe the properties of polygons (including triangles and quadrilaterals) using properties of shapes, including line and rotational symmetry	› identify, classify, and describe the properties of: – regular and irregular polygons, using edges, vertices, and angles – prisms, using the cross section, faces, edges, and vertices
	› compare and classify angles in 2D shapes equal to, smaller than, or larger than a right angle	› identify and describe parallel and perpendicular lines, including those forming the sides of polygons
	› use the mathematical processes to: – connect angles with turns – investigate the properties of triangles and polygons – investigate line and rotational symmetry – explain and justify whether lines are parallel and shapes are regular – explain and justify the value of unknown angles in triangles and quadrilaterals	
Spatial reasoning	› identify the 2D shapes that compose 3D shapes (e.g., a triangular prism is made up of two triangles and three rectangles)	› visualise and connect 3D shapes with their nets, their 2D diagrams, verbal descriptions of them, and the same shapes drawn from different perspectives
	› visualise, predict, and identify which shape is a reflection, rotation, or translation of a given 2D shape	› resize a 2D shape so that it is either bigger or smaller
	› use the mathematical processes to: – connect enlargement with simple grid references or coordinates and with doubling and halving – generalise the properties of shapes that do not change when transformed – investigate nets that fold together, shapes that tesselate, and transformations	

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing shapes:
› classify, identify, and explain similarities and differences between: – 2D shapes, including types of triangle – prisms and pyramids	› a range of 2D and 3D shapes, including tactile materials, diagrams, and digital tools
› identify and describe the interior angles of triangles and quadrilaterals	› angle benchmarks (year 4) › diagrams › protractors
› use the mathematical processes to: – connect angles with turns – investigate the properties of triangles and polygons – investigate line and rotational symmetry – explain and justify whether lines are parallel and shapes are regular – explain and justify the value of unknown angles in triangles and quadrilaterals	› notation › vocabulary (e.g., obtuse, acute, reflex, and right angles).
	Use the following when representing spatial reasoning:
› visualise and draw nets for rectangular prisms	› sketching and constructing 3D shapes
› visualise, create, and describe 2D geometric patterns and tessellations using rotation, reflection, and translation, and identifying the properties of shapes that do not change	› grids for resizing (year 5) › 2D shapes, squared paper, and tracing paper to predict and test transformations (years 4 & 6)
› use the mathematical processes to: – connect enlargement with simple grid references or coordinates and with doubling and halving – generalise the properties of shapes that do not change when transformed – investigate nets that fold together, shapes that tesselate, and transformations	› 3D shapes seen from different perspectives › grids.

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Pathways	› use grid references to identify regions and to plot positions on a grid map › interpret and describe pathways, including half and quarter turns and the distance travelled	› interpret and create a grid map to plot positions and pathways, using grid references and directional language, including the four main compass points
	› use the mathematical processes to: – connect compass points with angles and turns, and grid references with graphing skills – investigate different types of maps.	

Statistics

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Problem	› investigate summary and comparison situations with categorical and discrete numerical data, using multivariate (2 or more variables) data, by – posing summary and comparison investigative questions that can be answered with data – making predictions or assertions about expected findings	› investigate summary and comparison situations with categorical and discrete numerical data, using multivariate data by – posing summary and comparison investigative questions that can be answered with data – making predictions or assertions about expected findings
	› use the statistical processes to investigate school-related issues of interest	
Plan	› plan how to collect primary data to support answering an investigative question, including: – deciding on the group of interest – deciding the variable(s) for which data will be collected – taking account of ethical practices in data collection	

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
	Use the following when representing represent pathways:
› interpret and create grid references and simple scales on maps › use directional language, including the four main compass points, turn (in degrees), and distance (in m, km), to locate and describe positions and pathways	› maps with coordinates and compass points
› use the mathematical processes to: – connect compass points with angles and turns, and grid references with graphing skills – investigate different types of maps.	› graphs and maps on grids.

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› investigate summary, comparison, and time-series situations, using multivariate data to: – pose investigative questions that can be answered with data – make predictions or assertions about expected findings	Demonstrate posing investigative questions, and support students to write their own.
› use the statistical processes to investigate school-related issues of interest	Demonstrate the use of context analysis diagrams to represent school-related issues.
› plan how to collect primary data or how to use provided data, including identifying the variables of interest and, for provided data: – identifying who the data was collected from – identifying the original investigator's purpose for collecting the data – deciding if the source is reputable, by checking if any survey questions appear to be biased towards a particular point of view	Demonstrate asking evaluation questions about sources and ethical practices.

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Plan	› use the statistical processes to: – investigate topics of interest – explain and justify primary and secondary data, sensitive topics or questions, and ethical practices for data collection and use	
Data	› use a variety of tools to collect data, and check for errors in the data	› use a variety of tools to collect data, check for errors in the data, and correct errors by re-collecting the data, if possible
	› use the statistical processes to investigate methods for collecting secondary data	
Analysis	› create and describe data visualisations for summary and comparison investigations that make meaning from the data, with statements including the name of the variable	› create and describe data visualisations for summary and comparison investigations that make meaning from the data, with statements including the names of the variable and group of interest
	› use the statistical processes to: – investigate appropriate situations – explain and justify using 'I notice' statement about data visualisations, selecting the visualisation that best represents the data	
Conclusion	› choose the best descriptive statements to answer the investigative question, reflecting on findings and how they compare with initial predictions or assertions	› answer the investigative question, comparing findings with initial predictions or assertions and their existing knowledge of the world
	› use the statistical processes to: – connect statements with data visualisations to answer an investigative question, and to connect initial predictions or assertions with actual findings – investigate appropriate situations	
Statistical literacy	› check the statements that others make about data to see if they make sense, using information to clarify or correct statements where needed.	› check and, if needed, improve the statements others make about data, including data from two or more sources.
	› use the statistical processes to investigate, interpret, critique, and check the claims made about data presented in tables, pictographs, bar graphs, line graphs, and pie charts.	

Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› use the statistical processes to: – investigate topics of interest – explain and justify primary and secondary data, sensitive topics or questions, and ethical practices for data collection and use	Demonstrate how to represent data collection methods, who to measure, what to measure, and how to use a planning tool.
› describe information about variables in secondary data by using publisher-provided data dictionaries (e.g., how data was collected for them and possible outcomes for them)	Demonstrate a range of data collection methods and what errors in data 'look like'.
› use the statistical processes to investigate methods for collecting secondary data	Demonstrate data collecting, conducting observations, and designing surveys and questionnaires.
› create and describe a variety of data visualisations that make meaning from the data, identifying features, patterns, and trends in context, including the variable and group of interest	Demonstrate constructing and analysing data visualisations.
› use the statistical processes to: – investigate appropriate situations – explain and justify using 'I notice' statement about data visualisations, selecting the visualisation that best represents the data	Demonstrate representing data using dot plots, bar graphs, frequency tables, and time-series graphs.
› answer the investigative question, comparing findings with initial predictions or assertions and their existing knowledge of the world	Demonstrate making statements about data that match investigative questions.
› use the statistical processes to: – connect statements with data visualisations to answer an investigative question, and to connect initial predictions or assertions with actual findings – investigate appropriate situations	Demonstrate the use of concise statements.
› identify, explain, check, and, if needed, improve features in others' data investigations (e.g., survey questions, misleading information or statements).	Demonstrate identifying misleading data, matching data visualisations, and checking the claims of investigations.
› use the statistical processes to investigate, interpret, critique, and check the claims made about data presented in tables, pictographs, bar graphs, line graphs, and pie charts.	Draw on data visualisations from a variety of sources.

Probability

	Year 4 *Informed by formative assessment, teach students to:*	Year 5 *Informed by formative assessment, teach students to:*
Probability investigations	› engage in chance-based investigations with equally likely outcomes by: – posing investigative questions – anticipating what might happen – identifying possible outcomes for the investigative questions – generating all possible ways to get each outcome (a theoretical approach) or undertaking a probability experiment and recording the occurrences of each outcome – creating data visualisations for possible outcomes – describing what these visualisations show – finding probabilities as fractions – answering investigative questions – reflecting on anticipated outcomes	› engage in chance-based investigations, including those with not equally likely outcomes, by: – posing investigative questions – anticipating what might happen – identifying possible outcomes for the investigative questions – generating all possible ways to get each outcome (a theoretical approach) or undertaking a probability experiment and recording the occurrences of each outcome – creating data visualisations for possible outcomes – describing what these visualisations show – finding probabilities as fractions – answering investigative questions – reflecting on anticipated outcomes
Critical thinking in probability	› agree or disagree with others' conclusions about chance-based investigations.	› agree or disagree with others' conclusions about chance-based investigations, with justification.
	› use the statistical processes to: – connect the chance of an outcome occurring with fractions, decimals, and percentages – investigate everyday chance-based situations using physical activities and technology.	

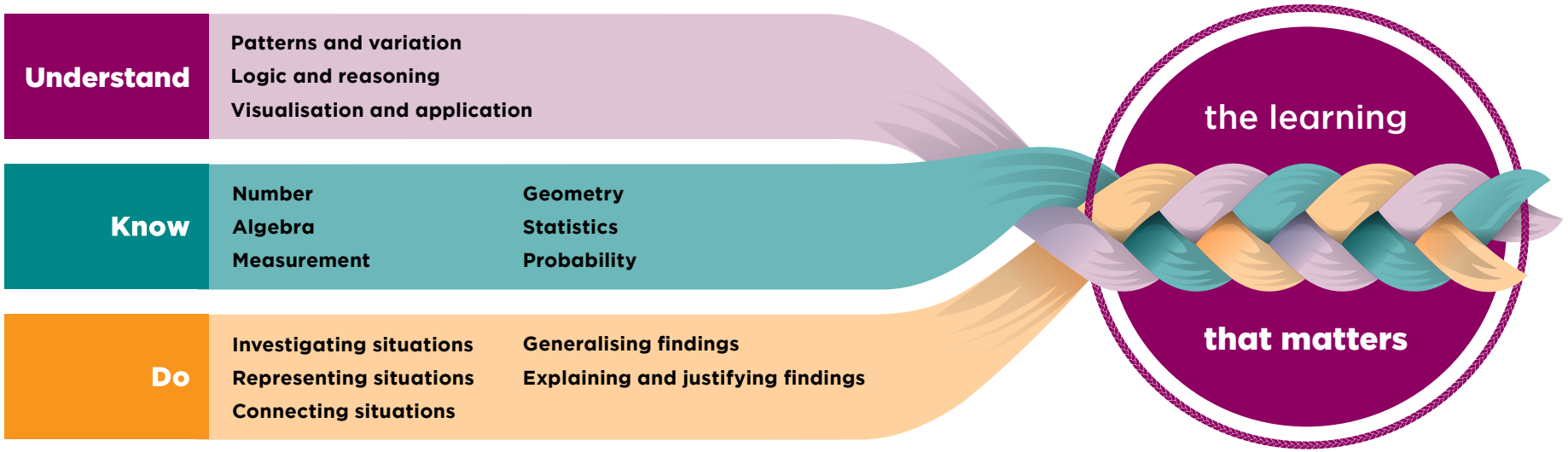
Year 6 ***Informed by formative assessment, teach students to:***	**Teaching methods**
› engage in one-stage, chance-based investigations, including those with not equally likely outcomes, by: – posing investigative questions – anticipating what might happen – identifying possible outcomes for the investigative questions – generating all possible ways to get each outcome (a theoretical approach) or undertaking a probability experiment and recording the occurrences of each outcome – creating data visualisations for possible outcomes – describing what these visualisations show – finding probabilities as fractions – answering investigative questions – reflecting on anticipated outcomes – comparing findings from probability experiments and associated theoretical probabilities, if the theoretical model exists	Play games of chance (e.g., Biased Bingo). Demonstrate creating systematic records of possible outcomes. Demonstrate using data visualisations.
› interrogate statements that others make about one-stage, chance-based situations, referring to evidence.	Demonstrate matching claims to data visualisations, using evidence from the data. Support students to interpret data and construct a response (e.g., with sentence starters, writing frames) (year 6).
› use the statistical processes to: – connect the chance of an outcome occurring with fractions, decimals, and percentages – investigate everyday chance-based situations using physical activities and technology.	Demonstrate representing outcomes using lists, tables, tree diagrams, tally charts, distributions, visualisations, words, and numbers.

Phase 3
Years 7–8

Seeing ourselves in the wider world and advocating with and for others
Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu

Progress outcome by the end of year 8

In phase 3, students weave together Understand, Know, and Do and share their logic and reasoning using maths notation, conventions, and vocabulary to clearly explain and justify their problem-solving approaches. They use number lines to visualise and extend their reasoning about positive integers to negative integers, enabling application to situations such as temperature change and financial transactions. They also use tables and graphs to represent linear patterns, and generalise by forming equations. They reason about unknown angles, explore variations in shapes through transformation, and use data visualisations to investigate claims and make predictions.



Understand
Big ideas

As they further develop their maths capabilities and knowledge, students use **patterns** and a multitude of relationships in which change and **variation** occur. They draw on the structures of mathematics and statistics to support noticing, exploring, and describing different types of patterns and relationships, generating insights and generalisations and making predictions. By engaging with maths concepts, students develop **logical** reasoning and critical thinking skills that enable them to critically evaluate information, question assumptions, and present arguments with clarity.

Statistical **reasoning** from observation and theory allows them to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable. Students further develop the ability to **visualise** maths ideas in order to perceive, understand, and interact with abstract concepts. They **apply** maths ideas to understand familiar and unfamiliar situations, facilitating better decision making and communication of ideas.

Know
Content and concepts

Number | Mātauranga tau

By the end of this phase, students know that some **numbers have special properties**, such as primes, composites, squares, square roots, and cubes. A fraction can describe a **proportional relationship** between two amounts. Every fraction can be represented by an infinite set of equivalent fractions that occupy the same point on the number line. Multiplying a fraction by an **equivalent form** of 1, such as $\frac{3}{3}$, results in an equivalent fraction that can be useful for comparing, adding, and subtracting. **Decimals** continue the place-value system using negative powers of ten. They can be terminating, repeating and infinite, or non-repeating and infinite.

Students know that **integers** are positive and negative whole numbers, as well as zero. On a number line, fractions and decimals occur between integers. There are real-life situations described by quantities less than zero, such as temperature, below sea level, or debt, and these quantities can be operated on.

Students know that when calculating or forming expressions, the **order of operations** is important. Operations inside brackets (grouped together) are done first, then powers or exponents. If there are multiplication and division, these are done in left-to-right order; addition and subtraction are also done in left-to-right order. Students use the mnemonic GEMA: grouped, exponents, multiplication, addition. Division can result in a **remainder** expressed as a whole number, fraction, or decimal.

Algebra | Taurangi

By the end of this phase, students know that the **inverse property** applies to addition and multiplication. **Inequalities** can also include "or equal to" (≤, ≥) to show a relationship that allows for the possibility of equality. In algebra, a **variable** can be used to represent an unknown number, a quantity that can vary or change (e.g., y= 3x + 4, A = bh), or a specific unknown value to be solved for (e.g., 3a = 18). In algebra, there are conventional ways of writing multiplication and division.

Students also know that **linear patterns** have a constant rate of change and their XY graphs are **straight lines**. Not all patterns are linear. **Algorithms** help solve problems in a systematic way. Their instructions are created, tested, and revised.

Measurement | Ine

By the end of this phase, students know that in the **metric system** there are **base** measurements with **prefixes** added to show the size of units. A measurement can be converted from smaller to bigger units, and vice versa, by dividing or multiplying by powers of 10. Length is a **one-dimensional measure**, area is a **two-dimensional measure**, and volume is a **three-dimensional measure**. This is apparent in the notation of units, such as cm, cm^2, and cm^3. Shapes can be **decomposed** or **recomposed** to help us find measurements (e.g., for perimeters, areas, and volumes). When multiplying lengths or dividing an area or volume by a length, the result has a **derived unit**.

Geometry | Āhuahanga

By the end of this phase, students know that **spatial properties** of simple **polygons** and **polyhedra** can also apply to more complex two- and three-dimensional shapes. Properties of two- and three-dimensional shapes that do not change under a transformation are called **invariant**. Unknown angles can be found using the properties of angles on a **straight line**, angles at a **point, vertically opposite** angles, and interior angles in triangles and quadrilaterals. Viewing objects from different angles gives **different perspectives**, which can be represented in models and diagrams. **Position**, **direction**, and **pathways** can be described using scale, compass points, and environmental features. **Coordinate systems** and maps can express position, direction, and pathways.

Statistics | Tauanga

By the end of this phase, students know that data collection and use involves a responsibility to protect the rights of people in relation to their own data and that of others, and the **ethical use** of data, including methods, interpretation, and conclusions. People need to understand who they are giving data to and why, before they agree to contribute to a dataset. The **statistical enquiry cycle** (PPDAC) can be used to conduct data-based investigations about the wider community. There are different types of **questions** used when undertaking statistical investigations: investigative (**summary, comparison, relationship, or time-series**), survey, data-collection, interrogative, or analysis questions. **Data visualisations** show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information. A **distribution** is formed from all the possible values of a variable and their frequencies. A **relationship** investigation looks for a relationship between paired numerical or paired categorical variables. **Predictions** or **assertations** may not be reflected in the data.

Probability | Tūponotanga

By the end of this phase, students know that a probability experiment involves repeated trials. Results from sets of repeated trials for the same probability experiment may vary. Some chance-based situations, such as the result of tossing a drawing pin, can only be explored by probability experiments. Estimates of probabilities from experiments should be based on a very large number of trials (the law of large numbers). The estimated probability of an event from an experiment equals the relative frequency for that event.

If all possible outcomes in a chance-based situation are **equally likely**, the probability of an event equals the number of ways the event can happen divided by the total number of possible outcomes. The statistical enquiry cycle (PPDAC) can be used to conduct experiments. For a given situation, **estimated probabilities** from **experiments** and **theoretical model** probabilities will differ. **Probability distributions** from experiments and probability distributions from theoretical models will also differ.

Do
Practices

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing a question, plan an investigation pathway, and follow it step by step. They can identify relevant prior knowledge, givens, and relationships to support the investigation. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

Students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisations to mentally represent and manipulate objects and ideas.

Connecting situations | Te tūhono pūāhua

Students can suggest connections between ideas, approaches, and different representations. They connect new ideas to things they already know. They make connections with ideas in other learning areas and with familiar cultural, linguistic, and historical contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

Students can recognise and explore patterns, and make conjectures and draw conclusions about them. They can identify relationships, including similarities, differences, and new connections. They look for patterns and regularities that might be applied in another situation or always be true. They make and test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

Students can make statements, give explanations inductively based on observations or data, and make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others, and they present reasoned explanations and arguments for an idea, solution, or process.

Phase 3
Years 7–8

Teaching sequence

Seeing ourselves in the wider world and advocating with and for others
Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu

Throughout phase 3, demonstrate, highlight, and affirm an attitude of exploration, enthusiasm, and curiosity towards maths endeavour and challenge, holding high expectations for every student. Students should critically reflect on others' reasoning, evaluating their logic and asking questions to clarify. To promote this, facilitate ongoing discussions and reflections about established expectations for interactions in maths learning, reinforcing that all students are involved. Support increasing agency for students to make decisions about investigations and problem solving (e.g., while planning their approach, selecting representations, and justifying their findings).

Continuously monitor students' reasoning, questions, and use of representations, and act quickly to respond to any misconceptions. Ensure your teaching builds on what students already understand, know, and can do.

Throughout each day, provide opportunities for students to:

- **develop positive relationships with maths –** provide authentic tasks that are meaningful to students' experiences, interests, and the wider world. Demonstrate and teach strategies for persistence (e.g., trying another way or drawing a diagram). Plan to balance teacher talk with opportunities for rich, extended student interactions and discussions
- **actively listen to, reflect, and build on each other's thinking and learning –** use discourse-based tools and a range of open questions to facilitate productive and challenging discussions. Over the phase, encourage students to convert their observations into a conjecture or claim and to use evidence to justify their claims and findings

- **experience explicit teaching of concepts and practices –** use worked examples and break down new learning into clearly explained manageable steps. Show students efficient written methods. Where there may be an error, misconception, or missing step, use examples to support students to develop critical analysis and reasoning
- **select and use representations that best support the learning purpose –** encourage students to use graphs, tables and equations. Over the phase, they will increasingly use equations to represent their reasoning. Visualising a situation by drawing a diagram can give students a way into a problem
- **learn and use maths vocabulary and concepts –** ensure students connect the correct vocabulary to the learning purpose and problem (e.g., by using the Frayer model's four quadrants: definition, characteristics, example, and non-example). Draw on students' first and heritage languages
- **use visualisation to represent and manipulate relationships, shapes, and quantities –** encourage students to use visualisation when, for example, predicting or deducing the effect of a transformation, viewing a solid shape from different perspectives, using coordinate pairs and locations, and identifying terms in a growing pattern
- **actively recall learning, practise new skills and processes, and make connections with prior learning –** provide regular opportunities to practise, so that students maintain their automatic recall of facts and continue to develop procedural fluency and reasoning. Support students to consolidate what they have learned by repeating the process or task you have demonstrated, firstly in familiar situations and then in unfamiliar situations.

Number

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
			Use the following when representing number structure:
Number structure	› identify, read, write, compare, and order whole numbers using powers of 10 (e.g., 10,000 = 10^4)	› identify, read, write, compare, and order whole numbers and decimals using powers of 10 (e.g., 0.01 = $\frac{1}{100}$ = 10^{-2})	› place-value (PV) houses, number lines, inequality symbols
	› find the highest common factor (HCF) of two numbers under 100, and find the least common multiple (LCM) of two numbers under 10	› use prime factorisation to represent a number and to find the HCF of two numbers	› factor trees, systematic lists
	› use exponents to notate repeated multiplication, and identify square roots of square numbers up to at least 100	› identify prime and composite numbers up to at least 100 and cube numbers up to at least 125	› divisibility tests for composite and prime numbers › demonstrations of exponent notation
	› use the mathematical processes to: – connect with divisibility rules, simplifying fractions, area, and volume – generalise conjectures about prime or composite numbers – investigate appropriate situations		› PV materials › factor trees › systematic lists.
			Use the following when representing operations:
Operations	› use rounding and estimation to predict and to check the reasonableness of calculations	› use rounding and estimation (including benchmarks) to predict and to check the reasonableness of calculations	› known facts › benchmarks
	› round whole numbers to any specified multiple of powers of 10, and round decimals to the nearest tenth, hundredth, or whole number		› number lines › visualising benchmarks

	Year 7 *Informed by formative assessment, teach students to:*	**Year 8** *Informed by formative assessment, teach students to:*	**Teaching methods**
Operations	› multiply whole numbers		› horizontal and vertical methods
	› divide whole numbers by 1- or 2-digit divisors (e.g., 327 ÷ 5 = 65.4 or 65 $\frac{2}{5}$)	› divide whole numbers (e.g., 327 ÷ 15 = 21.8 or 21 $\frac{4}{5}$)	› horizontal and vertical methods
	› use the order of operations rule GEMA	› use the order of operations rule GEMA	› demonstrations of step-by-step layouts
	› order, compare, add, and subtract integers using tools	› order, compare, add, and subtract integers	› number lines, two sided counters, diagrams
	› use the mathematical processes to: – investigate situations where integers are used (e.g., temperature, altitude, profit and loss) – explain and justify findings using estimation, and checking using inverse operations		› number lines.
			Use the following when representing rational numbers:
Rational numbers	› identify, read, write, and represent fractions, decimals (to three places), and percentages	› identify, read, write, and represent fractions, decimals, and percentages	› bar models, number lines
	› compare, order, and convert between fractions, decimals (to three places), and percentages	› compare, order, and convert between fractions, decimals, and percentages	› double number lines › benchmarks for demonstrating conversion › renaming to tenths or hundredths, or finding a common denominator
	› multiply and divide numbers by powers of 10		› PV houses
	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers		› demonstrations of simplifying fractions and finding equivalent fractions using division and multiplication
	› multiply fractions and decimals by whole numbers, and find a percentage of a whole number		› horizontal and vertical methods › demonstrations of finding a percentage using multiplication

	Year 7 ***Informed by formative assessment, teach students to:***	**Year 8** ***Informed by formative assessment, teach students to:***	**Teaching methods**
Rational numbers	› find a whole amount, given a simple fraction or percentage (e.g., '25% is $100, what is the original amount?')	› find a whole amount, given a simple fraction or percentage (e.g., '75% is $45, what is the original amount?')	› bar models
	› add and subtract fractions with different denominators up to tenths (e.g., $\frac{3}{4} + \frac{1}{3}$)	› add and subtract fractions with different denominators by using equivalent fractions	› renaming using HCFs and LCMs
	› add and subtract decimals to three decimal places, with an emphasis on estimating before calculating	› add, subtract, and multiply decimals, with an emphasis on estimating before calculating	› horizontal and vertical methods
	› use proportional reasoning to explore relationships between quantities (e.g., 'If there are 3 red for every 7 blue balls, how many balls are there altogether when there are 18 red balls?')	› use proportional reasoning to share in unequal proportions (e.g., 'We have 100 stickers to share. for every 1 sticker I get, you get 3 stickers. How many do we each get?')	› diagrams and comparison models
	› use the mathematical processes to: – connect benchmarks (fractions, decimals, and percentages) and decimal operations with whole-number place values and operations – connect decimals with measuring – investigate HCFs and LCMs, the effect of multiplying and dividing decimals, situations where decimals are used and compared (e.g., sporting events), and proportional reasoning – explain and justify equivalence and which fraction is larger		› continuous materials › bar models › written methods.
			Use the following when representing financial maths:
Financial maths	› calculate costs, and change for any amount of money	› create and compare weekly, monthly, and yearly finance plans (e.g., saving plans, phone plans, budgets, and 'buy now, pay later' services)	› spreadsheets › practical examples

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
Financial maths	› calculate the percentage discounts of whole dollar amounts (e.g., 'What is 35% of $180?')	› calculate percentage discounts	› demonstrations of finding a percentage and subtracting it from the whole, or of multiplying a whole by a required decimal fraction
	› use the mathematical processes to: – connect negative numbers with debt – investigate practical financial decisions and statistics in the media about growth or loss.		› digital tools (e.g., spreadsheets) › written and mental methods.

Algebra

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
			Use the following when representing number properties:
Generalising number properties	› explore multiplicative inverses (a number and its reciprocal) in multiplication		› fraction tiles, number lines › examples (for identifying trends)
	› explore additive inverses (pairs of opposites) in the addition and subtraction of positive and negative numbers (e.g., -6 + 8 = -6 + 6 + 2)	› use commutative, associative, identity, and inverse properties with expressions, including those with negative numbers	› number lines
	› recall multiplication facts to at least 10 × 10 and identify and describe the divisibility rules for 2, 3, 5, 9, and 10	› identify and describe the properties of prime and composite numbers and explore divisibility rules	› pattern exploration in 100s boards › multiplication grids

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
Generalising number properties	› describe and use the commutative, distributive, and associative properties of operations (e.g., O × __ = __ × O)	› simplify algebraic expressions involving sums, products, and differences, including by expanding single brackets expressions using the distributive property (e.g., 2(x + 3) + 1 = 2x + 6 + 1 = 2x + 7)	› physical manipulatives (e.g., algebra tiles) and digital manipulatives › systematic expansion approaches, including expansion tables
	› use the mathematical processes to: – represent algebraic expressions and equations using correct vocabulary and notation (e.g., 3 × b = 3b) – connect prime and composite numbers with factors, multiples, and divisibility rules – generalise relationships between positive and negative integers using the commutative, associate, and distributive properties of numbers – investigate appropriate situations		› algebraic notation.
			Use the following when representing equations and relationships:
Equations and relationships	› form and solve 1-step linear equations (e.g., t + 7 = 12 ; 2s = 14)	› form and solve 1- or 2-step linear equations (e.g., 5s – 3 = 17)	› word problems › demonstrations of equation-solving algorithms
	› find the value of an expression or formula given the values of variables (e.g., calculate w + 12 when w = 4)	› find the value of an expression or formula given the values of variables	› variable values in practical situations with familiar formulae (e.g., for area, volume)
	› identify the constant rate of change and fixed value for a linear pattern, writing the equation using variables and algebraic notation to represent the rule, and using the rule to make predictions	› determine if a pattern is linear and, if it is, write the equation for the pattern and use the equation	› tables and XY graphs › demonstrations of finding, and recording as an equation, the term-to-term and position-to-term rules for a sequence
	› use the mathematical processes to: – connect to measurement formulae – generalise a rule for a pattern and use this to justify a prediction of a term – investigate the history and use of growing patterns in tukutuku and other well-known patterns (e.g., the Fibonacci sequence)		› word problems › equations › tables and XY graphs.

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
			Use the following when representing algorithmic thinking:
Algorithmic thinking	› create, test, and revise algorithms involving a sequence of steps and decisions	› create, test, revise, and use algorithms to identify, interpret, and explain patterns	› flowcharts, numbered step-by-step instructions, digital tools
		› use the formula function of a spreadsheet to explore the effect of changing the value of a variable (e.g., hourly wages) on the results (i.e., cell values)	› spreadsheet cell conventions and formulae
	› use the mathematical processes to connect algorithms with methods for solving an operation.		› step by step instructions, diagrams, flow charts › variables on a spreadsheet.

Measurement

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
			Use the following when representing measuring:
Measuring	› estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate metric units		› rulers, scales, timers, protractors, thermometers, measuring jugs (always ensuring accurate use)
	› select and use an appropriate base measure (e.g., metre, gram, litre) within the metric system, along with a prefix (e.g., kilo, centi) to show the size of units		› practical measuring › units that are appropriate for the situation

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
Measuring	› convert between metric units of length, mass (weight), and capacity, using whole numbers and decimals to express parts of a unit (e.g., 724g = 0.724kg)	› convert between metric measurement units, including square units	› multiplying and dividing by powers of 10 › PV and decimal PV houses
	› find speed given distance and time	› find distance given speed and time, or time given distance and speed	› bar models showing relationships between speed, distance, and time
	› use the mathematical processes to: – generalise equivalent measurements (e.g., 2.05L = 2050mL) – connect measurement conversions with multiplying and dividing by powers of 10 – investigate practical measurement situations, including reading tools with scales – explain which measurement tools and units are appropriate in a given situation		› measurement tools › PV materials.
			Use the following when representing perimeter, area, and volume:
Perimeter, area, and volume	› calculate the perimeter and area of compound shapes composed of triangles and rectangles	› calculate the volume of triangular prisms and shapes composed of rectangular prisms	› demonstrations of finding perimeter, area and volume in practical situations › clear layouts for working
	› use the mathematical processes to: – generalise the formulae for finding the area of triangles and volume of triangular prisms – investigate practical contexts for finding perimeter, area, and volume		› practical measuring › written methods with clearly laid out working.
			Use the following when representing time:
Time	› read, interpret, and use timetables and charts that present measurement information		› demonstrations using a range of examples, including online apps
	› convert between units of time and solve duration problems that involve fractions of time		› subtracting time to calculate for duration › inclusive counting

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
Time	› use the mathematical processes to: – generalise units of time using base-60 – investigate the duration of time in situations such as developing event schedules or planning journeys.		› calendars, timetables, and schedules.

Geometry

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
			Use the following when representing shapes:
Shapes	› classify shapes based on their properties, and name the resulting classes of shapes (e.g., triangles, pyramids)	› describe triangles, quadrilaterals, and other polygons in relation to their side, diagonal, and angle properties	› a range of 2D and 3D shapes, including shapes that draw on tactile materials, diagrams, and digital tools
	› identify and describe angles at a point, angles on a straight line, and vertically opposite angles	› reason about unknown angles in situations involving angles at a point, angles on a straight line, vertically opposite angles, interior angles of triangles, and polygons	› digital tools for exploring angles › measuring tools (e.g., rulers, protractors)
	› use the mathematical processes to: – generalise using angle rules to find unknown angles – investigate diagonals and angles of polygons – explain and justify classifications using flowcharts, Venn diagrams, and tables		› equations (to find unknown angles).

	Year 7 *Informed by formative assessment, teach students to:*	Year 8 *Informed by formative assessment, teach students to:*	Teaching methods
			Use the following when representing spatial reasoning:
Spatial reasoning	› visualise, construct, and draw plan views for front, back, left, right, and top views of 3D shapes, using cube models, digital tools, and grid paper	› visualise and draw nets for prisms with a fixed cross section	› physical models or sketches, created using measurement tools
	› transform 2D shapes, including composite shapes, by resizing by a whole number or unit fraction of less than one	› recognise the invariant properties of 2D and 3D shapes under different transformations	› resizing using a centre of enlargement › 2D shapes and squared paper › tracing paper to predict and test transformations
	› use the mathematical processes to: – investigate the meaning of kowhaiwhai patterns and other symbols from te ao Māori, and describe the use of transformations in these patterns – explain which properties of a shape will be affected by a given transformation		› drawings and models › nets.
			Use the following when representing pathways:
Pathways	› interpret and communicate the location of positions and pathways using coordinates, angle measures, and the 8 main and halfway compass points (e.g., 45° E from N is NE)	› use map scales, compass points, distance, and turn to interpret and communicate positions and pathways in coordinate systems and grid reference systems	› demonstrations using 4-digit grid references and scale to calculate actual distance
	› use the mathematical processes to: – connect map scales to proportional reasoning – connect angles and using a protractor with compass points – investigate the most efficient route between two destinations.		› maps of familiar and unfamiliar locations › protractors and angle notation › diagrams of compass points › coordinate and grid references.

Statistics

<table>
<tr><th></th><th>Year 7
Informed by formative assessment, teach students to:</th><th>Year 8
Informed by formative assessment, teach students to:</th><th>Teaching methods</th></tr>
<tr><td rowspan="2">Problem</td><td colspan="2">› investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations for paired categorical data by:
– posing investigative questions about local community matters
– making predictions or assertions about expected findings</td><td rowspan="2">Demonstrate posing investigative questions, and support students to write their own questions.</td></tr>
<tr><td colspan="2">› use the statistical processes to:
– represent summary, comparison, relationship, and time-series investigative questions
– investigate a broad area of interest before fine-tuning a specific investigative question</td></tr>
<tr><td rowspan="2">Plan</td><td colspan="2">› plan how to collect or source data to answer investigative questions, including
– determining or identifying the variables needed
– planning how to collect data for each variable (e.g., how to measure them when collecting) or finding out how provided data was collected
– identifying the group of interest or who the data was collected from
– building awareness of ethical practices by strategic questioning of data collection methods</td><td rowspan="2">Demonstrate making a plan for how to collect and sort the data needed to answer an investigative question.</td></tr>
<tr><td colspan="2">› use the statistical processes to:
– represent using a diagram who, what, and how to measure
– investigate appropriate situations
– explain and justify variables and groups of interest when working with secondary data</td></tr>
<tr><td rowspan="2">Data</td><td>› collect data, including
– checking for errors, following up and correcting them when possible
– creating data dictionaries that include information for others about the context</td><td>› source ready-to-use data, and provide information about the variables using provided data dictionaries</td><td rowspan="2">Demonstrate a range of data collection methods and what errors in data ‘look like’.</td></tr>
<tr><td colspan="2">› use the statistical processes to:
– represent data using a range of tools (e.g., spreadsheets, recording sheets)
– investigate secondary data
– explain errors in data and justify why they are errors</td></tr>
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Informed by formative assessment, teach students to:</th><th>Teaching methods</th></tr>
<tr><td rowspan="2">Analysis</td><td>› create and describe data visualisations for summary, comparison, relationships (paired categorical), and time-series investigations, including features and context in descriptions of distributions</td><td>› create and describe data visualisations for summary, comparison, relationships, and time-series investigations, using multiple visualisations to provide different views of the data and including features and context in descriptions of distributions</td><td rowspan="2">Demonstrate constructing and analysing data visualisations.</td></tr>
<tr><td colspan="2">› use the statistical processes to:
– represent data using dot plots, bar graphs, frequency tables, time-series graphs, two-way tables or graphs, scatter plots, fractions, proportions, and percentages
– investigate how different data visualisations show different features of data and give different information
– explain and justify patterns, trends, and features of data visualisations</td></tr>
<tr><td rowspan="2">Conclusion</td><td>› communicate findings in context to answer an investigative question, using evidence from analysis and comparing findings to initial predictions or assertions and existing knowledge of the world</td><td>› communicate findings in context to answer an investigative question, using evidence from analysis, considering possible explanations for findings, and comparing findings to initial predictions or assertions and existing knowledge of the world</td><td rowspan="2">Demonstrate making statements about data and matching them with investigative questions.
Demonstrate reflecting on findings.</td></tr>
<tr><td colspan="2">› use the statistical processes to:
– connect statements with data visualisations to answer an investigative question
– investigate appropriate situations
– explain findings, and justify initial predictions or assertions given the findings</td></tr>
<tr><td rowspan="2">Statistical literacy</td><td>› examine the findings of others to check if their claims or statements are supported by the data visualisations they use</td><td>› examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are reasonable</td><td rowspan="2">Demonstrate identifying misleading data, matching data visualisations, and checking the claims of investigations.</td></tr>
<tr><td colspan="2">› use the statistical processes to explain and justify critiques of data visualisations and collection methods.</td></tr>
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Probability

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Informed by formative assessment, teach students to:</th><th>Year 8
Informed by formative assessment, teach students to:</th><th>Teaching methods</th></tr>
<tr><td>Probability investigations</td><td colspan="2">› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using technology, by:
– posing investigative questions
– identifying outcomes for the investigative question posed and anticipating what might happen
– deciding on the number of trials, the tools to be used, and the recording method
– collecting and recording data
– creating data visualisations for the distribution of observed outcomes and (year 8) for all possible outcomes for theoretical probability models where they exist
– describing what these visualisations show
– finding the probability estimates for the different outcomes
– proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists
– identifying similarities and differences between their findings and those of others
– reflecting on anticipated outcomes
– identifying similarities and differences between findings from probability experiments and associated theoretical probabilities, as appropriate</td><td>Use probability experiments such as taking coloured blocks out of a bag, drawing a card, and flipping a coin, comparing theoretical with experimental probability.
Demonstrate creating data visualisations that show outcomes.</td></tr>
<tr><td rowspan="3">Critical thinking in probability</td><td colspan="2">› agree or disagree with others' conclusions by interrogating their probability experiments</td><td rowspan="3">Demonstrate and support students to interpret data and construct a response, using sentence starters or writing frames.
Use 'true or false' or 'odd one out' challenges, asking students to support their ideas with reasons.</td></tr>
<tr><td></td><td>› agree with or challenge claims and identify misconceptions in relation to chance-based situations</td></tr>
<tr><td colspan="2">› use the statistical processes to:
– represent outcomes using systematic approaches and technology
– connect probabilities with proportional reasoning, fractions, and percentages
– investigate games of chance, patterns in possible outcomes, and theoretical and experimental distributions
– explain and justify probability estimates and claims about chance-based situations.</td></tr>
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