Te Tāhuhu o te Mātauranga
Ministry of Education

Te Poutāhū
Curriculum Centre

Wāhanga Ako
Pāngarau

Te Kāwanatanga o Aotearoa
New Zealand Government

Direction for school boards about requirements

Kura and schools must give effect to the Pāngarau Wāhanga Ako Years 0–8

The *Pāngarau Wāhanga Ako Years 0–8* is published by the Minister of Education, Hon Erica Stanford, under section 90(1) of the Education and Training Act 2020 as a foundation curriculum policy statement and a national curriculum statement.

The sections which are made as national curriculum statements are Whenu and Tohu Ako (excluding Mā te Kaiako). These set out what students are expected to learn over their time at school, including the desirable levels of knowledge, understanding, and skill to be achieved.

The rest (including Mā te Kaiako within the Tohu Ako section) is made as foundation curriculum policy statements. These set out expectations for teaching, learning and aromatawai that underpin the national curriculum statements and give direction for effective Pāngarau teaching and learning programmes.

The statements came into force on **1 January 2025** and replace the 2010 Pāngarau national curriculum statement (wāhanga ako) **for students in years 0–8**. Other existing national curriculum statements for *Te Marautanga o Aotearoa (2010)* remain in place.[1]

These are the statements of official policy in relation to the teaching of Pāngarau that give direction to each school's curriculum and aromatawai responsibilities (section 127 of the Education and Training Act 2020) (the Act), teaching and learning programme (section 164 of the Act), and monitoring and reporting of student performance (section 165 of the Act and associated Regulations). As required under these sections of the Act, school boards must make sure that their school's principal and staff develop and implement teaching and learning programmes that give effect to these statements.

What is required?

Kura and schools must be able to demonstrate that they have used these statements when planning what and how to teach Pāngarau for students in years 0–8. This includes teachers:

- using the year-by-year teaching sequence in Tohu Ako to inform what to teach and when, based on their students' prior learning
- including all the pedagogical guidance and teaching strategies, including Mā te Kaiako and Te Roanga o te Kōrero for kaiako in their teaching practice
- using aromatawai to ascertain their students' progress and achievement for Pāngarau based on the Tohu Ako progress outcomes.

Teachers should continue to use their professional judgement to adapt their teaching and learning programmes to their students' needs - which may mean that some students learn at a different part of the teaching sequence than their year level. If students need extension beyond year 8 for the Pāngarau wāhanga ako, teachers should use the Curriculum Levels 5 and above in the 2010 Pāngarau national curriculum statement (wāhanga ako).

Pānui, tuhituhi, and pāngarau teaching time requirements[2]

The teaching and learning of pānui, tuhituhi, and pāngarau is a priority for all schools. So that all students are getting sufficient teaching and learning time for pānui, tuhituhi, and pāngarau, each school board with students in years 0 to 8 must, through its principal and staff, structure their teaching and learning programmes and/or timetables for delivering the National Curriculum Statements, including this one, to provide:

- 10 hours per week of teaching and learning focused on supporting their progress and achievement in pānui and tuhituhi in a typical school week, recognising the important contribution oral language development makes, particularly in the early phases of learning.
- 5 hours per week of teaching and learning focused on supporting their progress and achievement in pāngarau in a typical school week.

Where pānui, tuhituhi, and/or pāngarau teaching and learning time is occurring within the context of National Curriculum Statements other than Pāngarau (years 0–8, 2024) or Pāngarau (2010 curriculum, level 5 up), progression of students' pānui, tuhituhi, and/or pāngarau dispositions, knowledge and skills at the appropriate level must be explicitly and intentionally planned for and attended to.

While the terms pānui and tuhituhi are used, these expectations are inclusive of alternative methods of communication, including New Zealand Sign Language, augmentative and alternative communication (AAC) and Braille.

[1] https://gazette.govt.nz/notice/id/2009-go8814

[2] https://gazette.govt.nz/notice/id/2023-go5904

Pāngarau

Tēnā koutou katoa,

It is my pleasure to introduce the revised Pāngarau wāhanga ako for Years 9-13 in Te Marautanga o Aotearoa. This marks the first time we have developed a national curriculum that is knowledge-rich, year-by-year, sequenced, and based on the science of learning. Specific subjects for Pāngarau for Years 11, 12 and 13 are easily identified.

This new curriculum ensures clarity about what students need to understand, know, and do each year, while also providing teachers with a clear framework, resources and supports to guide their classroom practice.

Our aim is to reduce ambiguity, empowering educators to teach with confidence, and ensuring that all students have access to a high-quality education that prepares them for success in the future. This curriculum reflects the dedication of many individuals working together to offer our young people the best educational opportunities.

Pāngarau serves as a bridge between the Māori world and the world of mathematics, positioning the mokopuna at the beginning of a learning journey that fosters a successful future. The mathematical principles and processes ground a person in their identity while connecting them to the broader world.

Pāngarau provides opportunities for mokopuna to engage with important societal matters, such as the robust and ethical gathering, interpretation, and communication of data, and the use of valid and reliable data to exemplify information, progress, achievements and opportunities important to them, their whānau, hapū, iwi, and hapori.

Ultimately, this new content is about raising achievement and closing the equity gap. Every ākonga should have the opportunity to realise their full potential and thrive in both the classroom and in life, no matter their background or where they attend kura.

I encourage you to engage with this wāhanga ako and provide your feedback so that we can ensure your expertise as classroom teachers and experts in mathematics education is reflected in the final version of this wāhanga ako, due for release later this year. Every piece of feedback is incredibly valuable, and I look forward to working together to ensure that this curriculum is truly world-leading and sets up both our teachers and students for success.

Ngā mihi nui,

Hon Erica Stanford (Minister of Education).

Te Iho o Pāngarau

'Kei hopu tāu ringa ki te aka tāepa,
engari kia mau ki te aka matua'

Te Iho

Pāngarau serves as a bridge between the Māori world and the world of mathematics, positioning the mokopuna at the beginning of a learning journey that fosters a successful future. The mathematical principles and processes ground a person in their identity while connecting them to the broader world.

Purpose

This learning area fosters mokopuna who are empowered to contribute holistically to their communities. Mokopuna learn to weave language, dispositions, processes, knowledge, and skills of pāngarau with their mathematical counterparts in ways that are logical, purposeful, and usable across mathematical knowledge areas. This supports mokopuna to explore, understand and participate in their world locally, nationally, and globally.

Pāngarau allows us to engage with important societal matters, such as the robust and ethical gathering, interpretation, and communication of data, and the use of valid and reliable data to exemplify information, progress, achievements and opportunities important to them, their whānau, hapū, iwi, and hapori. Kaiako, whānau, hapū, iwi and hapori support mokopuna to make beneficial future decisions by connecting their pāngarau learning to other areas where mathematics is useful, including other wāhanga ako, and career pathways.

Teaching and learning of pāngarau has a deliberate focus on ensuring mātauranga, tikanga, and kaupapa Māori perspectives of whānau, hapū, iwi, and hapori contribute to what is taught and learnt. Te reo, mātauranga Māori and kaupapa a-iwi provide the foundations for developing pāngarau teaching and learning, whilst encouraging mokopuna to be critical thinkers. The underlying idea is that kaiako and mokopuna grow a love for pāngarau.

Pāngarau Processes

Mathematicians use pāngarau processes to make sense of situations and to solve problems. Kaiako and whānau support is an integral part of helping mokopuna to confidently engage with mathematical knowledge and processes, particularly for their futures. Mokopuna use mathematical processes to learn and apply mathematical knowledge and skills. They also support mokopuna to connect mathematical knowledge, language, procedures, and rules with their environment, with ever changing technologies, and guide them into specialist mathematical career pathways and industries. Investigating situations, problem-solving, and communicating have strong links to all the other pāngarau processes.

Pāngarau processes reflect tīpuna ways of knowing, being and doing. Mokopuna use these processes to understand and solve problems based on both theoretical and real-world scenarios. Like their tīpuna, mokopuna develop their ability to confidently draw conclusions, communicate findings and justify their reasoning while analysing situations from different perspectives.

By working with their peers, mokopuna reflect on their own thinking and the thinking of others, consciously adjusting their strategies to improve the accuracy and efficiency of their findings. Mathematical processes cannot be separated from the knowledge and skills that mokopuna acquire. Mokopuna must problem solve, communicate, reason, and reflect, as they develop the knowledge, the understanding of concepts, and the skills required in all the strands at any age[1].

[1] *New Math Curriculum for Grades 1-8.* (June 2020). Government of Ontario. https://www.ontario.ca/page/new-math-curriculum-grades-1-8

Pāngarau Processes				
Investigating Situations	**Representing Situations**	**Connecting Situations**	**Generalising Findings**	**Explaining and Justifying Findings**
• Pose a question for investigation. • Find entry points for addressing a question, identifying relevant prior knowledge, facts, and relationships. • Plan an investigation pathway and follow it step by step. • Monitor and evaluate progress, adjusting the investigation pathway where necessary. • Make sense of findings.	• Use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. • Use representations to learn new ideas and explain ideas to others. • Investigate conjectures, and support arguments. • Select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. • Use visualisation to mentally represent and manipulate objects and ideas.	• Suggest connections between ideas and approaches. • Suggest connections between different representations. • Connect new ideas to prior knowledge. • Make connections with ideas in other learning areas and in familiar cultural, linguistic, and historical contexts.	• Recognise and explore patterns, make conjectures, and draw conclusions about them. • Identify relationships, including similarities, differences, and new connections. • Look for patterns and regularities that might be applied to another situation or always be true. • Make and test conjectures, using reasoning and counter examples to decide if they are true or not. • Use appropriate symbols to express generalisations.	• Make statements and give explanations based on observations or data. • Make statements and give explanations based on knowledge, definitions, and rules. • Critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. • Use evidence, reasoning, and proofs to explain agreement or disagreement with statements. • Develop collective understanding by sharing, comparing, contrasting, critiquing, and building on ideas with others. • Present reasoned explanations and arguments for an idea, solution, or process.

Perspectives

Pāngarau provides mokopuna with a range of perspectives, tools, and processes to encourage critically informed decision-making; supporting mokopuna to actively contribute to their whānau and communities as Māori and global citizens with mana motuhake.

Pāngarau views the teaching and learning of mathematics in the following ways:

Whakapapa

Pāngarau is connected to whakapapa, reflecting the depths of ancestral knowledge and historical information that has been handed down from generation to generation.

Tūrangawaewae

Pāngarau is a bridge linking us to our cultural identity, reminding us that mathematics is enhanced when grounded in the place we call home and the connections we share with our ancestors and community.

Mana Motuhake

Pāngarau provides opportunities for mokopuna to engage with mathematical practices that reflect their values and aspirations.

Kaitiakitanga

Pāngarau encourages us to be stewards of knowledge that is both a taonga and a vital resource, emphasising our responsibility to protect and sustain the practices that allow us to understand and manage our world effectively.

Whanaungatanga

Pāngarau fosters the creation of a shared framework of knowledge and problem-solving that strengthens relationships and collaboration.

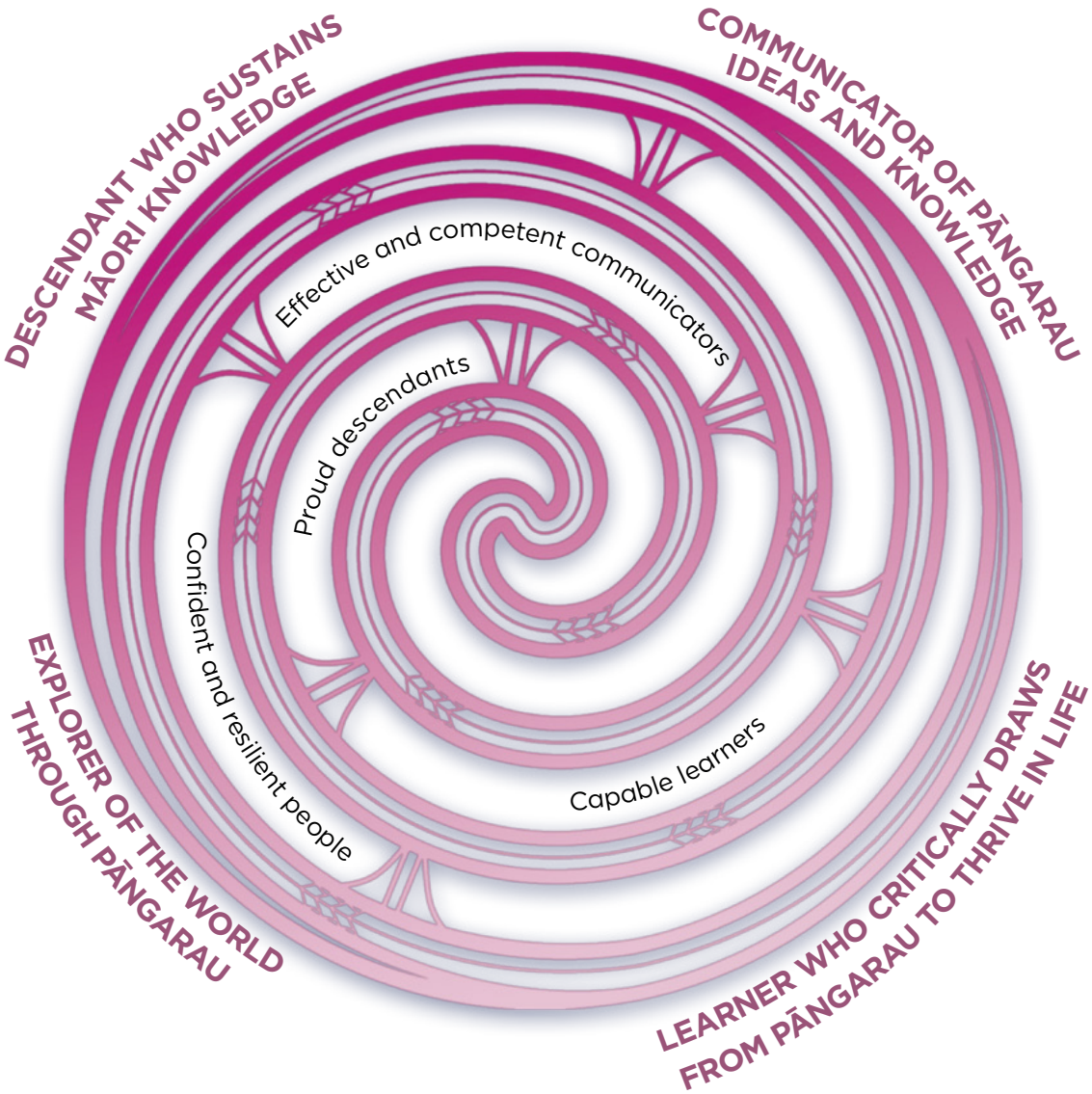
Structure

Strands

Pāngarau has four strands:

- Explorer of the world through pāngarau | HE TANGATA tūhura ki te ao.
- Descendant who sustains mathematical knowledge from the past, into the present and onto the future | HE URI WHAKAHEKE ki te whai ao.
- Communicator of pāngarau ideas and knowledge | HE PUNA KŌRERO o te reo pāngarau.
- Learner who critically draws from pāngarau to thrive in life | HE ĀKONGA mauri oho.

These strands contribute to the **Pou Matua** and the four outcomes of the curriculum.



He Tangata tūhura ki te ao

Mokopuna develop their approaches and skills using a pāngarau investigative process that becomes more sophisticated as they develop their dispositions and knowledge. They will learn to identify situations where pāngarau can support them to plan mathematical investigations and/or solve problems. They will learn how to ask good mathematical questions and plan with close support from their kaiako until they gradually demonstrate movement towards the independence necessary to move into the wider world.

He Uri Whakaheke ki te whai ao

Mokopuna focus on how mathematics tikanga, mātauranga and reo can support them to maintain and sustain tikanga, mātauranga and reo Māori (and vice-versa) when they learn and use pāngarau.

He Puna Kōrero o te reo pāngarau

Mokopuna investigate and draw from their inherent knowledge, helping them to represent, model, discuss and demonstrate what they are doing mathematically.

He Ākonga mauri oho

Mokopuna use pāngarau content knowledge to investigate and think about situations and/or problems where mathematics is helpful and beneficial for the future aspirations of the mokopuna, their whānau, hapū and iwi.

Toi Mokopuna

Toi Mokopuna describe the dispositions and attributes of learners studying pāngarau. They reflect the aspirations of whānau, hapū, and iwi, the desired outcomes of learning pāngarau, and the rationale for each strand. The dispositions remain the focus of learning across the phases and provide clarity for teachers and learners about the purpose of the knowledge, skills, understandings and learning experiences embedded in each strand.

Strands	HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Toi Mokopuna	Mokopuna are flexible, interested, curious, collaborative, creative and reflective investigators, willing to test strategies to solve problems for the school, themselves, their whānau, hapū and iwi.	Mokopuna are agile thinkers like their ancestors. Pāngarau ways of thinking and doing have been passed to them and will be passed on to others.	Mokopuna are confident and articulate in their reasoning, explaining and communicating mathematical ideas and concepts, and are interested in the role of mathematics and its value as a different type of language.	Mokopuna are persistent and reflective, monitoring and adjusting their own thinking and performance by contextualising and identifying patterns and connections, constructing mathematics and their worlds.

A Progression of Pāngarau Learning

Pāngarau shows a progression of development across five phases from Year 0 to Year 13. At each phase there are four key objectives, one per Toi Mokopuna. The key objectives signal the focus of learning at that phase and have been determined according to the most significant learning required at each phase.

Each mokopuna learns at their own pace and in their own way. Therefore, it is important that kaiako are familiar with not only the year they teach, but also the focus of learning in years before and ahead. This is one of the advantages of a phase spanning multiple years; kaiako can easily see the focus of learning in the previous years and/or in the years ahead. This also reflects the reality of multi-level classrooms where mokopuna are at multiple stages of learning.

Key Objectives

At each phase, four key objectives are identified:

	HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Learning focuses on:				
Phase 1 (Years 0-3)	developing skills for conducting mathematical investigations and understanding how mathematics can be applied to situations in their daily lives.	developing an understanding of basic mathematical expressions and concepts in relation to themselves and how it influences their world.	developing their mathematical language to communicate basic expressions and concepts.	developing skills to identify patterns and connections, by drawing, building, copying, and making representations of existing patterns.
Phase 2 (Years 4–6)	carrying out mathematical investigations that align with their world and developing strategies that can be applied to their daily lives.	using mathematics as a resource to support their problem-solving activities in situations that relate to their world.	understanding and using a range of mathematical expressions and concepts to report on their findings.	understanding, making and explaining relationships, patterns, models, and representations related to mathematical processes and knowledge.
Phase 3 (Years 7-8)	choosing a topic for mathematical investigation and justifying the methods chosen to solve problems and evaluate situations.	using mathematics to complete projects for themselves, the kura and the community.	justifying outcomes using the correct mathematical expressions and concepts in alignment with and across each strand.	building and presenting models showing relationships and patterns that explain the mathematical knowledge being used.
Phase 4 (Years 9-10)	carrying out project-based investigations and exploring mathematical techniques for analysing and solving problems. Evaluating solutions and reporting on situations using mathematical knowledge.	understanding how the global application of mathematical outcomes supports evidence-based decision-making to benefit mokopuna, whānau, hapū, iwi and their communities. This process helps mokopuna explore options for their future, the future of others, including their communities, and their wellbeing.	explaining outcomes and evidence using increasingly complex mathematical language across the focus areas.	evaluating mathematical models and representations to understand mathematical conjecture, proofs, and generalisations.
Phase 5 (Years 11–13)	carrying out project-based investigations and justifying the mathematical techniques selected for analysing and solving problems. Evaluating solutions and situation reports through mathematical knowledge.	understanding the global application of mathematics for the wellbeing of whānau, hapū, iwi, and hapori; using mathematical outcomes as evidence to guide future options and directions locally, nationally, and globally.	using specialised mathematical language to communicate investigative findings into specific areas, explaining the evidence to both mathematical experts and other interested audiences.	evaluating mathematical models and representations to gain insight into conjectures, hypotheses, and generalisations.

Unpacking the Key Objectives

Pāngarau objectives are broken down in terms of what needs to be taught at each year level within a phase.

Tohu Ako describe the yearly mathematical content to be learnt beginning from the end of the first 6 months at kura to year 3 in Tūārere 1. Tūārere 2 covers content from years 4 to 6. Tūārere 3 covers years 7 and 8. Tūārere 4 covers years 9 and 10 and Tūārere 5 covers years 11 to 13. Yearly content knowledge is developed according to mathematical knowledge progressions and grouped according to Toi Mokopuna.

Pāngarau learning is an ongoing, cyclical process with varying degrees of progress being made at different times. Mokopuna need repeated practice at new skills to become competent. Therefore, all significant learning is developed through each phase. However, kaiako need to explicitly focus on the skills, dispositions, knowledge, and understandings, signalled at a specific year.

Where there is a critical point of learning that kaiako must be aware of, this is stated in **Kia Mataara.** Not all whenu, or phases will include **Kia Mataara** learning. However, where **Kia Mataara** learning is signalled, kaiako must take particular care in the teaching and learning of these aspects as they can affect the future learning and achievement of mokopuna.

Te Ngako o te Whāinga (unpacking the whāinga) provides a summary of key learning aligned to the key objectives in relation to Toi Mokopuna. In this section, teaching and learning is described in year levels so that it is clear what the focus is each year. This provides guidance for kaiako about what the big ideas of mathematics are and what to do to best support mokopuna learning. This includes integrating Toi Mokopuna with mathematics by aligning with the four pillars. Although the progressions are yearly, kaiako discretion to notice, recognise, and respond appropriately will be useful to recognise when mokopuna show they are capable of the work involved in the ngako and/or when they have already surpassed that knowledge and should be drawing from the next set of whāinga is important.

The full detail of an objective is unpacked in **Te Roanga o te Kōrero** (full explanation). The finer aspects to be learnt and the teaching required is explained in this section. The section is specific to the phase and reflects the reality of teaching and learning more accurately, where mokopuna learn at different rates. Te Roanga o te Kōrero starts with a general description of mokopuna as learners and what should be expected during this phase, as well as what kaiako need to consider in terms of mokopuna development.

Whāinga are further elaborated on and kaiako are given specific guidance about what they will be doing in terms of Ako. Characteristics of mokopuna and what they will do as pāngarau learners, and what will be nurtured in mokopuna to support them to be confident and competent learners of mathematics are described.

Specific unpacking is defined in terms of the following key elements: strategies and skills, knowledge, language, and identity signalling the breadth of learning embedded in a key objective and what needs to be taught. Referred to as **Ngā Tini Mata o te Ako** (the many aspects of teaching and learning), these elements reflect the key ideas of an Ako pedagogy and espouse that all kaiako are teachers of learning, of language and of mokopuna.

In the section, **Hei Tautoko i te Ako** (supporting teaching and learning), the pedagogical considerations are explained including suggested teaching strategies, resources, and exemplar language.

Kaiako, mokopuna and whānau are also provided with the progressions through mathematical strands up to Year 13. Key knowledge, processes and skills are identified but not grouped under **Toi Mokopuna**.

Diversity in Pāngarau Learning

Recognising the diverse learning styles of all mokopuna is critical for knowing how to facilitate learning pāngarau. Start by speaking with whānau to identify and understand mokopuna unique strengths and challenges and what works for them. Use Te Ngako o te Whāinga as the basis for a discussion on their priorities for pāngarau development.

Become knowledgeable about strategies that work for neurodiverse and physically diverse mokopuna and what these look like in a pāngarau learning context.

During Pāngarau lessons engage all mokopuna by:

Being Consistent

- having a consistent structure so mokopuna know what to expect
- maintaining momentum and flow through the lesson
- name the strategies you use so mokopuna know what to do each time.

Focusing on Communication

- speak clearly, and succinctly – give short instructions
- ensure mokopuna do most of the talking – teacher talk time is specific.

Fostering Active Participation

- engage mokopuna with communicative tasks and strategies that are fun
- encourage mokopuna to use whatever and all language they know to complete pāngarau learning tasks
- encourage collaboration among mokopuna
- encourage and celebrate “having a go” with using new mathematical concepts.

Promoting Learning

- use a range of specific pāngarau teaching methods appropriate to mokopuna year level, sequence, and stage of learning to identify what works for each mokopuna
- provide scaffolded support for mokopuna who have diverse learning needs
- start with what mokopuna already know as the basis for learning new mathematical concepts
- use familiar contexts and strategies when introducing new mathematical concepts
- teach mokopuna strategies for learning new mathematical words and concepts
- utilise visual, auditory, and tactile methods to engage different learning styles
- include regular time for reflection to help mokopuna identify the strategies that work for them and questions they may have
- foster an inclusive classroom environment that values all learning differences.

Monitoring and Aromatawai

National monitoring practices and qualifications data provide a view of pāngarau achievement across Years 0–13.

In kura, aromatawai practices are also used to monitor individual mokopuna progress, enabling kaiako to celebrate the achievements of each mokopuna. By setting clear priorities and specific goals, kaiako can better understand and support the growth of every mokopuna.

The information gathered through aromatawai practices should inform kura, kaiako, and whānau about the effectiveness of the pāngarau programme. It should also give mokopuna and kaiako a clear understanding of what has been achieved and the progress made.

Ako is the Foundation of Aromatawai

Monitoring and measuring pāngarau development should not "look" any different to pāngarau teaching and learning. The difference is in what, and how, the kaiako notices what mokopuna are doing. Active observation is a key strategy for monitoring progress:

1. Pay close attention to what the mokopuna is doing or communicating – observe, listen, notice, reflect.
2. Recognise specific knowledge, skills, attitudes, and behaviours the mokopuna is demonstrating.
3. Reflect and think carefully about what the mokopuna can do and how to build on from that.

Aromatawai is Mana-Enhancing

Pāngarau aromatawai and monitoring should focus on identifying the pāngarau knowledge, skills, dispositions, and language that a mokopuna has mastered in order to know how effective the teaching has been and what the next steps should be.

In order to acknowledge the full breadth and diversity of learning, kaiako and kura gather evidence of progress across all whenu and areas of pāngarau including:

- knowledge (ngā tini mata o te mātauranga)
- skills (ngā tini mata o te ako)
- dispositions (ngā tini mata o te tuakiri)
- language (ngā tini mata o te reo).

Aromatawai is Evidence-Based

Kaiako and kura are responsible for the judgements they make about mokopuna and their pāngarau development. It is essential that kaiako, kura, whānau, and mokopuna all have confidence in the evidence used to support these decisions. To ensure that judgements about pāngarau learning are both reliable and valid, a combination of tangible and intangible processes are used.

Tangible processes include learning interactions, formal monitoring, and informal practices. Intangible processes involve more nuanced approaches, such as using senses and deeply understanding the mokopuna as an individual – their needs, strengths, challenges, and dispositions.

The **Te Poutama Tau diagnostic framework** models the kinds of processes that kaiako can use to understand what mokopuna know, how they talk about that knowing, and what they do to show their knowledge. Two other available resources for aromatawai are:

- He Pūkete Aromatawai Pāngarau
- He Tauaromahi Pāngarau

Tūārere 4
Years 9–10

Whenu			
HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Toi Mokopuna			
Mokopuna are flexible, interested, curious, collaborative, creative and reflective investigators, willing to test strategies to solve problems for the school, themselves, their whānau, hapū and iwi.	Mokopuna are agile thinkers like their ancestors. Pāngarau ways of thinking and doing have been passed to them and will be passed on to others.	Mokopuna are confident and articulate in their reasoning, explaining, and communicating mathematical ideas and concepts, and are interested in the role of mathematics and its value as a different type of language.	Mokopuna are persistent and reflective, monitoring and adjusting their own thinking and performance by contextualising and identifying patterns and connections, constructing mathematics and their worlds.
Tohu Ako: Phase 4: Years 9–10			
Whāinga			
Mokopuna learning focuses on carrying out project-based investigations and exploring mathematical techniques for analysing and solving problems. Evaluating solutions and reporting on situations using mathematical knowledge.	**Mokopuna learning focuses on** understanding how the global application of mathematical outcomes supports evidence-based decision-making to benefit mokopuna, whānau, hapu and iwi. This process helps mokopuna explore options for their future lives and wellbeing.	**Mokopuna learning focuses on** explaining outcomes and evidence using increasingly complex mathematical language across the focus areas.	**Mokopuna learning focuses on** evaluating mathematical models and representations to understand mathematical conjecture, proofs, and generalisations.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 4: Years 9–10			
Kia Mataara			
By the end of year 9, mokopuna: • find a variable in an aspect of something that changes or can be changed and, in an investigation, collect data about one or more variables related to the subject being studied.	**By the end of year 9, mokopuna:** • plan how to use their mathematical knowledge to support the achievement of tasks of importance to their whānau • plan how to relate their mathematical knowledge to mātauranga Māori.	**By the end of year 9, mokopuna:** • analyse a data presentation to determine the validity and accuracy of the conclusions drawn.	**By the end of year 9, mokopuna:** • measure and partition all types of numbers and understand that the total of the parts is equal to the whole of the measurement • use a variety of complex strategies to solve problems involving whole numbers, fractions, decimals, percentages, indices, and integers (positive and negative numbers) • use different units of measurement and compare these units.
By the end of year 10, mokopuna: • pose investigative questions about a wider population • critically evaluate statistical investigations and multivariate data • implement statistical experiments and gather sufficient data to develop experimental results • demonstrate their understanding of the relationship between experimental results and theoretical models.	**By the end of year 10, mokopuna:** • demonstrate their ability to apply mathematical knowledge to achieve tasks of importance to their whānau, and wider communities • demonstrate their ability to think critically about how mathematical knowledge relates to mātauranga Māori.	**By the end of year 10, mokopuna:** • explain, extend, and expand patterns and connections in all areas of mathematics • communicate their findings with appropriate displays and generalise in context.	**By the end of year 10, mokopuna:** • graph linear relationships and form equations to describe relational representations • recognise the complexity of the attribute being measured and use correct units • solve measurement problems by splitting shapes into component parts, applying formulae, and using measurement ratios and trigonometric approaches appropriately • describe and manipulate vectors • use properties of angles formed by intersecting lines (like alternate interior angles) to determine relationships between lines • demonstrate how shapes can change while maintaining properties like congruence and similarity • apply Pythagoras' theorem in both 2D and 3D scenarios, reinforcing the relationship between algebra and geometry.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 4: Years 9–10			
Te Ngako o te Whāinga			
Mā te Kaiako			
Year 9 **Support mokopuna to:** • use the characteristics of decimal numbers including terminating, repeating, and infinite as well as non-repeating and infinite (irrational numbers) • solve fraction problems using the characteristics of fractions • define characteristics of prisms and use them to describe what they are and how they are formed • use the statistical enquiry cycle (PPDAC) to conduct data-based investigations involving population sampling.	**Year 9** **Support mokopuna to:** • demonstrate awareness that people have rights and obligations in relation to their own data and that of others • recognise that people have rangatiratanga over their data, including inherent rights and interests in relation to its collection, use, storage, and ownership • recognise that different countries have different laws about data and privacy.	**Year 9** **Support mokopuna to:** • use theoretical model probabilities and experimental estimates of probabilities to show how they are approximations of the true probabilities, which are unknown.	**Year 9** **Support mokopuna to:** • demonstrate that they can use time measurement systems based on 12–hour and 24-hour clocks correctly, and identify the difference between the two time systems • recognise where and how decimal measures are used for very small durations (milliseconds) • demonstrate their understanding that multiplying a fraction by an equivalent form of 1 (e.g., ½ x 3/3 = 1/2) results in an equivalent fraction • apply the general result for the sum of internal and external angles of polygons to solve problems • apply the properties of operations (e.g., inverse property, distributive property, commutative property, and identity property) to numbers and variables • use the order of operations when using numbers and variables • demonstrate how, in similar shapes, corresponding angles are equal and lengths of corresponding sides are proportional • demonstrate their understanding that angles between parallel lines and a transversal have known relationships (corresponding, alternate, or co-interior angles).

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 4: Years 9–10			
Te Ngako o te Whāinga			
Mā te Kaiako			
Year 10 **Support mokopuna to:** • sample from a population, the distribution for a variable that varies from sample to sample.	**Year 10** **Support mokopuna to:** • resize a shape and change its perimeter, area, and volume, demonstrating understanding of its connections • create new variables by combining and modifying existing variables • work on two-stage chance-based situations, using their understanding that the stages can be dependent (leading to conditional probabilities) or independent.	**Year 10** **Support mokopuna to:** • express functions as algebraic expressions, XY graphs, tables, or in words • use a range of different, equivalent equations to express a linear function.	**Year 10** **Support mokopuna to:** • use their knowledge that, in right-angled triangles, there is a fixed relationship between the lengths of the three sides given by Pythagoras' theorem • use their knowledge to identify that a point has zero dimensions, a line has one dimension, a plane is two-dimensional, and a solid is three-dimensional. In mathematics, there can be more than three dimensions • show that dividing by a divisor less than 1 gives a result bigger than the dividend • use rates to compare two quantities that have different units of measurement • use ratios to compare two like quantities • show how algorithms can be efficient or inefficient and that more efficient algorithms have fewer steps.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 4: Years 9–10			
Te Ngako o te Whāinga			
Mā te Mokopuna			
I am learning to: • find fractions or percentages of a number • compare fractions that arise from division-as-sharing (e.g., sharing 2 among 5 vs sharing 3 among 7) • simplify fractions using the highest common factor • use rates to model and represent change • use and apply ratios to model everyday situations • operate on numbers with whole number exponents • develop the rule for multiplying or dividing numbers with exponents and the same base • reason about unknown angles in situations involving parallel lines, transversals and the interior and exterior angles of polygons • indicate functions arising from linear and simple quadratic patterns • graph linear functions and interpret the gradient, x-intercept, and y-intercept in relation to the function or the practical situation represented • substitute into, rearrange, and simplify expressions, combining like terms as needed • create or use a formula, rule, equation, or inequality to solve for unknowns, and evaluate by substitution • identify how sequence, selection, and iteration are used in algorithms for generating patterns.	**I am learning to:** • accurately estimate, calculate, and represent measurements using significant figures • derive and use the formula to calculate area, perimeter and/or circumference of polygons and circles • scale a shape by a factor and derive the scale factor for the scaled shape's area or volume • plan probability experiments that use real data to create probability distributions for numerical variables, run simulations, and record data.	**I am learning to:** • pose summary and comparison questions about populations • pose investigative questions for time-series and relationship data and for experiments • plan data collection for observational studies and experiments, including selecting valid and reliable measurements for variables or sourcing existing datasets • re-categorise variables if needed and use multiple representations to analyse and visualise data • communicate findings using evidence from analysis • provide possible explanations for findings • reflect on predictions or assertions • critique the findings and claims of others by interrogating all phases of the statistical enquiry cycle • recognise and pose investigative questions, anticipate what will happen, and state assumptions • describe probability distributions including those involving simple, joint, and conditional probabilities • identify why claims about chance-based situations might not be valid and consider improvements to how the claim was investigated.	**I am learning to:** • add and subtract fractions with different denominators using equivalent fractions • multiply and divide two integers, two fractions, or two decimals • use the properties of similarity in two-dimensional shapes, including right-angled triangles, to find unknown lengths • use and apply Pythagoras' theorem to find the length of an unknown side in a right-angled triangle • use invariant properties to transform a set of points in the XY plane by translation, reflection about an axis, and rotation about a given point by a multiple of 90 degrees • represent three-dimensional shapes with two-dimensional drawings and digital tools • find the surface area, volume and/or capacity of prisms and cylinders.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 4: Years 9–10			
Te Roanga o te Kōrero			
Characteristics of Mokopuna Mokopuna use mathematical investigations to understand specific mathematical concepts, ideas, situations, and problems. Mokopuna can critique the mathematics they are using. They can also use mathematical tools and literacies to critique situations and solutions to problems that are important in their worlds. They are encouraged to use models, representations, words, symbols, and diagrams to both show and deepen their understanding of the work they are doing. Mokopuna understand that mathematical tools and solutions must be used and interpreted ethically.	**Characteristics of Mokopuna** Mokopuna develop critical use of mathematics in their everyday lives and can discern the similarities and differences in te ao Māori and te ao pāngarau cultural views. Mokopuna recognise that mathematics is useful for their purposes and what they want to do in their lives. They also recognise the value of mathematics for realising the aspirations of their whānau, hapū, iwi, and hapori. Mokopuna will think about local and global issues (wicked world problems) and how mathematics can support them to effect positive change at a local and global level.	**Characteristics of Mokopuna** Mokopuna ask appropriate, focused, and relevant mathematical questions about their worlds. Mokopuna can use a range of mathematical literacies to communicate their explorations and findings. Mokopuna can discuss their investigations and findings as part of mathematics communities. They know how to interact collaboratively and respectfully with others to discuss and debate the efficacy of their mathematical findings with a range of audiences. Their use of mathematical literacies supports them to evaluate and reflect on the world around them. Mokopuna understand that they need to use different forms of mathematical language when they are communicating with diverse groups of people or for different purposes.	**Characteristics of Mokopuna** Mokopuna can recognise different kinds of patterns, describe attributes, find themselves in space, think about possibilities and dimensions and connect this knowledge to higher levels of mathematical understanding and operations. Mokopuna use logic, mathematical reasoning and generalising to make sense of situations and to help solve problems. Mokopuna are able to articulate mathematical cultural tikanga, that differs from Māori culture and tikanga, and utilise each for rich purposes. Mokopuna can use and see use for mathematics in their hapori. Mokopuna make connections across mathematical areas that extend into other learning areas. Mokopuna are becoming more confident in their ability to make conjectures, and search for proofs.
Affirming Identity Mokopuna awareness of how others work things out, what others want to know about, and how to relate appropriately during mathematical investigations is increasing. This is part of their mathematical investigation culture as they work both independently and collaboratively. Mokopuna are creative with pāngarau, and critical thinkers using pāngarau.	**Affirming Identity** Mokopuna are demonstrating increasing confidence with mathematics culture, language and tikanga, and can articulate which parts of mathematical processes are based on tīpuna ways of thinking. They can also recognise when mathematical content knowledge is useful but does not supersede tīpuna knowledge and tikanga.	**Affirming Identity** Mokopuna confidence to communicate about their mathematical ideas and findings grows as they take on more sophisticated mathematical content knowledge. This is particularly relevant if they have meaningful reasons to share and communicate the findings of their mathematical investigations to wider audiences.	**Affirming Identity** Mokopuna are demonstrating further application of mathematical and cultural knowledge for their own purposes, identifying ways that they can use mathematical findings to inform decisions in their everyday lives.

Tohu Ako: Phase 4: Years 9–10

Number

Learning Strategies

Mokopuna are learning to:

- use a range of strategies to solve whole numbers and simple decimals including the use of inverse operations
- make use of a variety of additive strategies to solve problems involving whole numbers, fractions, decimals and integers
- estimate and solve complex problems for multiplication and division that involve whole numbers, decimals, fractions, and integers
- apply proportional thinking to solve real problems involving percentages, rates, and ratios, and to form one- and two-step algebraic equations to solve for an unknown.

The main strategies for the mokopuna to use are:

- broad strategies to solve addition and subtraction problems (use multiples, averages, and equivalent operations for negative numbers)
- solving decimal problems using additive inverse properties (6.03 - 5.8 = 5.8 + ? = 6.03) and calculating inverses (? + 3.98 = 7.04 = 3.98 + ? = 7.04)
- multiplying and dividing whole numbers, decimal numbers, and fractions by partitioning, using tidy numbers, compensation, proportional relationships and basic facts
- dividing quantities into ratios
- applying ratios and proportional relationships to solve problems.

Key Understandings

Number Structure

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• record, compare and order whole and decimal numbers using scientific notation: e.g., 2.45×10^3. • identify special properties of numbers, including cube roots of cube numbers to 1,000.	• record, compare and order whole and decimal numbers using scientific notation, including negative powers: e.g., 7.35×10^{-3} compared to 0.00735.

Operations

Mokopuna are learning to:	Mokopuna are learning to:
• use rounding and estimation to predict and check the reasonableness of calculations • express remainders as fractions and decimals depending on the context • add and subtract positive and negative numbers • multiply integers using repeated addition or subtraction • divide integers by reversing a multiplication.	• use rounding and estimation to predict and check the reasonableness of calculations • add, subtract, multiply and divide positive and negative numbers, including fractions and decimals.

Rational Numbers

Mokopuna are learning to:	Mokopuna are learning to:
• identify, read, write, represent, compare, order, and convert between fractions, decimals, and percentages.	• identify, read, write, represent, compare, order, and convert between fractions, decimals, and percentages.
• find equivalent fractions • simplify fractions • convert between improper fractions and mixed numbers.	• find equivalent fractions • simplify fractions • convert between improper fractions and mixed numbers.
• find fractions or percentages of a number • increase or decrease a number by a fraction or percentage.	• calculate the increase or decrease between numbers as a percentage change.
• find the whole amount, given a fraction or percentage: e.g., 20% of an amount is 30, what is the original amount?	• find the whole amount, given a fraction or percentage: e.g., 15% of an amount is 27, what is the original amount?
• divide fractions by whole numbers, and add, subtract, and multiply fractions • divide decimals by whole numbers, and add, subtract, and multiply decimals.	• add, subtract, multiply, and divide fractions and decimals.
• use ratios to share in unequal proportions • find equivalent ratio and rates by either scaling up or down.	• compare and use ratios and rates.

Tohu Ako: Phase 4: Years 9–10

Number

Financial Maths

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• calculate profit, loss, and discounts as absolute quantities relative to the original amount.	• convert NZ dollars into other currencies (and vice versa) and give examples of when this is useful.
• calculate simple interest and GST on dollar amounts: e.g., 15% of $432.	• find proportions of costs: e.g., price of 400g when given the cost per kilogram.

The Language of Number

Mokopuna know the following words:	Mokopuna know the following words:
• mixed numbers • original amount • unequal proportion • inverse proportion • equivalent ratio • positive / negative number • profit • loss • interest • discount • GST • absolute value • standard form.	• mixed numbers • rate • proportional problem • convert dollars • currencies.

Tohu Ako: Phase 4: Years 9–10

Algebra

Learning Strategies

Mokopuna are learning to:

- use diagrams, tables, graphs, and equations to solve problems that involve linear relationships
- graph linear relationships and demonstrate an understanding of a standard parabola
- form equations to describe relational representations and solve real problems.

Key Understandings

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• operate on numbers with whole number exponents • generalise the rule for multiplying or dividing numbers with exponents and the same base • express functions arising from linear and simple quadratic patterns • graph linear functions and interpret the gradient • find the x-intercept and y-intercept in relation to the function or practical situation represented • substitute into, rearrange, and simplify expressions, combining like terms as needed • create or use a formula, rule, equation, or inequality to solve for unknowns, and evaluate by substitution • identify how sequence, selection, and iteration are used in algorithms for generating patterns.	• operate on numbers with whole number exponents • generalise the rule for multiplying or dividing numbers with exponents and the same base • express functions arising from linear and simple quadratic patterns • graph linear functions and interpret the gradient • find the x-intercept and y-intercept in relation to the function or practical situation represented • substitute into, rearrange, and simplify expressions, combining like terms as needed • create or use a formula, rule, equation, or inequality to solve for unknowns, and evaluate by substitution • identify how sequence, selection, and iteration are used in algorithms for generating patterns.

Generalise Number Properties

Mokopuna are learning to:	Mokopuna are learning to:
• identify connection between adding and subtracting positive and negative numbers.	• identify connection between adding and subtracting positive and negative numbers.
• explore multiplicative inverses with negative numbers: e.g., 1/b = -1/b.	• identify how multiplication and division as inverse operations extend to negative numbers: e.g., 3 × -2 = -6.
• explore all operations including square roots with exponents.	• operate on numbers with whole number exponents, including exploring the effect of powers on negative numbers.
• simplify, expand, and factorise algebraic expressions involving sums, products, differences, and positive integer powers.	• simplify, expand, and factorise algebraic expressions.

Equations and Relationships

Mokopuna are learning to:	Mokopuna are learning to:
• form and solve linear equations with rational number solutions and linear inequalities with positive coefficients • find the value of an expression or formula given values of the variable and rearrange simple formulae.	• form and solve linear equations and inequalities with rational number solutions, giving exact and rounded solutions.
• form an equation with variables and algebraic symbols to show a formula using predictions.	• use an equation of linear function to draw graphs.
• form, interpret, graph, and express linear functions in the form y = mx + c, where 'm' is the constant rate of change (gradient or slope), and 'c' is the y-intercept.	• form, interpret, graph, and express linear functions in more than one form.
• recognise the relationships between elements in a non-linear pattern • write the equation using variables and algebraic notation to represent the rule • use the rule to make predictions, including quadratic patterns of the form, 'c' is an integer: e.g., y = x2 + c.	• determine the effect on XY graphs of changing the coefficient of x squared and the fixed value for a range of quadratic functions of the form y = ax squared, or y = x squared + c, where a is a positive whole number and c is an integer.

Tohu Ako: Phase 4: Years 9–10

Algebra

Algorithmic Thinking

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• create graphs of linear functions, write the equation to describe the relationship and use to solve problems • use the formula function of a spreadsheet to explore the effect of changing one or more variables on the results (cell values).	• choose the equation of a linear graph and explain the connection to two variables of the graph • use the formula function of a spreadsheet to investigate situations.

The Language of Algebra

Mokopuna know the following words:	Mokopuna know the following words:
• selection • linear relationships • axis • exponents • square root • expressions • function • same base • interpret • gradient • positive integer • negative integer • positive coefficient • multiplicative inverse • non liner pattern • parabola • spreadsheet.	• factorise • linear • function • expression • quadratic relation • integer • constant value • spreadsheet.

Tohu Ako: Phase 4: Years 9–10

Measurement

Learning Strategies

Mokopuna are learning to:

- deduce and use formula to measure perimeter, area, and volume
- understand that the relationship between circumference and diameter (or radius) is a constant ratio, and understand the metric relationship of 1 g = 1 ml = 1 cm^3.

Key Understandings

Measurement and Measuring

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• accurately estimate, calculate, convert, and represent measurements using significant figures.	• accurately estimate, calculate, convert, and represent measurements using significant figures.
• select and use an appropriate unit of measurement for the context, including converting between metric units using appropriate prefixes.	• convert between metric units and use the appropriate prefixes in the metric system: e.g., tera, giga, mega, kilo, centi, milli, micro, nano.
• use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle from a diagram.	• use and apply Pythagoras' theorem to find the length of an unknown side in a right-angled triangle.
• find speed, distance, and time from 2 measurements, recognising that distance over time is a derived measure.	• find speed, distance, and time from 2 measurements, recognising that distance over time is a derived measure.

Perimeter and Volume

Mokopuna are learning to:	Mokopuna are learning to:
• find the area of parallelograms, trapeziums, and kites and relate the formula to rectangles • find the circumference and area of circles.	• find the surface area and volume or capacity of prisms and cylinders.
• scale a shape by a factor and find the perimeter, area, or volume of the scaled shape.	• scale a shape by a factor and determine the scale factor for the scaled shape's area or volume.

Time

Mokopuna are learning to:	Mokopuna are learning to:
• make sense of and explain how cycles of patterns relate to the time indicators, such as the stars, seasons of the year, bird migrations and environmental signs throughout the year • reason about duration using different units of time and fractions of units of time, including decimal fractions (milliseconds) where appropriate.	• make sense of and explain how cycles of patterns relate to the time indicators, such as the stars, seasons of the year, bird migrations and environmental signs throughout the year • reason about duration using different units of time and fractions of units of time, including decimal fractions (milliseconds) where appropriate.

The Language of Measurement

Mokopuna know the following words:	Mokopuna know the following words:
• perimeter • area • volume • parallelogram • rhombus • trapezium • kite • scale • suffix • rotation.	• capacity • surface area • prism • cylinder • prefix • scale factor • season.

Tohu Ako: Phase 4: Years 9–10

Geometry

Learning Strategies

Mokopuna are learning to:

- name, identify and draw polygons using their two-dimensional and three-dimensional attributes
- design nets using different views, isometric diagrams, and scale
- describe coordinates, give directions using grid points, compass points, and scale, and use angles to show direction
- understand symmetry
- design nets using different views, isometric features and scale, and workout angle sizes
- follow a transformation sequence
- use ratio to calculate scale factors for enlargement.

Key Understandings

Shapes

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• identify and describe parts of circles and how they relate to each other: e.g., chord, diameter, radius, and circumference.	• use the properties of similarity in 2D shapes, including right-angled triangles, to find unknown lengths and angles.
• find unknown angles in situations involving intersecting, parallel lines, and transversals.	• find unknown angles in situations involving parallel lines and transversals and the interior and exterior angles of polygons.

Spatial Reasoning

Mokopuna are learning to:	Mokopuna are learning to:
• represent and construct 3D shapes, including rectangular and triangular prisms, from nets, plan views, or isometric diagrams.	
• transform 2D shapes in the XY plane by translation, reflection in a given mirror line, and rotation about a given point by a multiple of 90 degrees.	• resize by any scale factor to transform the size of 2D shapes, including composite shapes.
• use map scales and compass directions (e.g., N30°W) to interpret and communicate distance and direction between locations.	• interpret points and lines on co-ordinate planes, including scales and bearings on maps.
• define and use transformations and describe the invariant properties of figures and objects under these transformations.	• define and use transformations and describe the invariant properties of figures and objects under these transformations.

Co-ordinates and Direction

Mokopuna are learning to:	Mokopuna are learning to:
• use map scales to identify location points in the world: e.g., interpret and communicate distance and direction between 8 locations • use signs of nature to determine direction, such as the sun, winds and stars.	• use map scales to identify location points in the world: e.g., interpret and communicate distance and direction between 8 locations • use signs of nature to determine direction, such as the sun, winds and stars.

Tohu Ako: Phase 4: Years 9–10	
Geometry	
The Language of Geometry	
Year 9 **Mokopuna know the following words:**	**Year 10** **Mokopuna know the following words:**
• location points • symmetrical • grid point • scale factor of enlargement • mirror line • enlargement • parallel line • intersecting line • transversal line • diameter • radius • circumference • diagram • isometric diagram • rectangular prism • triangular prism • compass • scale • transformation • reflection • rotation • invariant properties.	• scale factor of enlargement • polygon • parallel line • transversal line • right-angled triangle • scale factor • exterior angle • interior angle • co-ordinate plane • bearings/direction • interpret.

Tohu Ako: Phase 4: Years 9–10

Statistics

Learning Strategies

Mokopuna are learning to:
- devise a statistical investigation enquiry that includes multivariate data. Create and interpret a range of data displays and evaluate relationships between variables.

Key Understandings

Plan and Make Sense of the Problem

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• plan how to collect or source data to answer investigative questions, including: - identifying the variables needed to answer the investigative question - planning how to make valid and reliable measures for the variables (when collecting) or finding out how they were collected (when sourcing) - identifying the group of interest or who the data was collected from - using a set of interrogative questions to check the different ethical practices that should be considered through the entire statistical enquiry cycle, including checking data collection and survey questions before testing with peers.	• plan how to collect or source data to answer investigative questions for observational studies and experiments, including: - identifying the variables needed to answer the investigative question - planning how to make valid and reliable measures for the variables (when collecting) - interrogating sourced datasets to understand what the variables measure and how they were measured - determining if the investigative question is about a group or a population, if it is a population, a sample from the population is taken to answer the investigative question - determining the sample size needed for categorical and numerical data to make a call about what is happening in the population - identifying the group or population of interest or who the data was collected from - using a set of interrogative questions to check ethical practices that should be considered throughout the entire statistical enquiry cycle.

Data

Mokopuna are learning to:	Mokopuna are learning to:
• determine appropriate variables and data collection methods.	• determine appropriate variables and data collection methods.
• gather, sort, and display multivariate category, measurement, and time-series data to detect patterns, variations, relationships, and trends.	• source data • make decisions about the validity of data and make simple edits if appropriate • find the metadata.

Analysis/Conclusion

Mokopuna are learning to:	Mokopuna are learning to:
• evaluate statements made by others about the findings of statistical investigations and probability activities.	• evaluate statistical investigations or probability activities undertaken by others, including data collection methods, choice of measures, and validity of findings.
• use multiple displays, and re-categorise data to find patterns, variations, relationships, and trends in multivariate data sets.	• use multiple displays, and re-categorise data to find patterns, variations, relationships, and trends in multivariate data sets.
• identify relevant features in distributions • interweave the context in the description of the distribution.	• identify relevant features in distributions • draw a line of best fit to predict possible values for the response variable for a given explanatory variable, where appropriate in relationship situations.

Tohu Ako: Phase 4: Years 9–10	
Statistics	
The Language of Statistics	
Year 9 **Mokopuna know the following words:**	**Year 10** **Mokopuna know the following words:**
• interrogative questions • testing • trends • distributions • ethics • time-series • patterns • variations • relationships • investigation method • probability.	• categorical data • numerical data • statistical enquiry cycle • conjecture • population • interview • distinctive features • ethics • source • focus group.

Tohu Ako: Phase 4: Years 9–10

Probability

Learning Strategies

Mokopuna are learning to:

- examine the steps in the process
- examine results of others' statistical investigations in order to evaluate the appropriateness of the methods and conclusions
- investigate and experiment with situations involving chance
- use and interpret models that show probability.

Key Understandings

Year 9 Mokopuna are learning to:	Year 10 Mokopuna are learning to:
• plan and conduct probability experiments for chance-based situations, including undertaking trials using technology, by posing investigative questions: - identifying outcomes for the investigative question posed and anticipating what might happen - constructing a model using theoretical probabilities or conjectured probabilities - deciding the number of trials, tools to be used, and recording method - running simulations and recording data - creating data visualisations for the distribution of observed outcomes from probability experiments and the distribution of possible outcomes for theoretical probability models - describing what these visualisations show - finding the probability estimates for the different outcomes - proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists: e.g., tossing a drawing pin - identifying similarities and differences between their findings and those of others - reflecting on anticipated outcomes - identifying and giving potential reasons for, similarities and differences between their findings from probability experiments and associated theoretical probabilities.	• plan and conduct probability experiments for chance-based situations, including undertaking trials using technology, by posing investigative questions: - identifying outcomes for the investigative question posed and anticipating what might happen - identifying model assumptions and limitations for the chance-based situation - constructing a model using theoretical probabilities, probabilities based on past data, or conjectured probabilities - deciding the number of trials, tools to be used, and recording method - running simulations and recording data - creating data visualisations for the distribution of observed outcomes from probability experiments and the distribution of possible outcomes for theoretical probability models - describing what these visualisations show - finding the probability estimates for the different outcomes - proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists: e.g., tossing a drawing pin - identifying similarities and differences between their findings and those of others - reflecting on anticipated outcomes - identifying, and giving potential reasons for, similarities and differences between their findings from probability experiments and associated theoretical probabilities.

Critical Thinking in Probability

Mokopuna are learning to:	Mokopuna are learning to:
• identify why claims about chance-based situations might not be valid and consider improvements to how the claim was investigated • evaluate the reasonableness of others' chance-based claims.	• identify why claims about chance-based situations might not be valid and consider improvements to how the claim was investigated • critically consider possible scenarios by using probability characteristics to: - identify the assumptions in the conclusions of probability models that have been used - evaluate the validity of the probability conclusions made by others - explain if there is misinformation and clarify why it is considered misleading.

Tohu Ako: Phase 4: Years 9–10	
Probability	
The Language of Probability	
Mokopuna know the following words:	**Mokopuna know the following words:**
• conjecture • outcome • model • theoretical probabilities • conjectured probabilities • trials • visualisations • findings • chance-based situations • chance-based claims • probability situation • probability experiments • associated probablities • theoretical model.	• simulation • data visualisations • limit • findings • outcome • output • conclusions • theoretical outcomes • theoretical model • associated probabilities • probability situation.

Hei Tautoko i te Ako

Pedagogical Guidance

Ensure the learning is accessible to all mokopuna

Kaiako are teachers of mokopuna, they are teachers of language and they are teachers of learning.

Design learning for all mokopuna by:

- providing opportunities to solve problems (e.g., ask critical questions, plan activities that prompt discussion, evaluation and assessment of critical issues relevant to mokopuna)
- using prompts (e.g. mathematical terms, previous learned concepts, diagrams, and materials, including use of digital tools)
- adapting the task (e.g. applying mathematical strategies and concepts into context, and gradually increasing the complexity)
- asking questions to redirect thinking or encourage connections
- chunking learning into smaller parts
- presenting ideas using different representations or materials
- providing opportunities to present findings from mathematical investigations to a wider audience
- further explicit teaching (reteaching), demonstrating, and consolidating
- changing the context to one that is more familiar to the mokopuna.

Support mokopuna with scaffolds such as:

- tailored teaching of specific mathematical skills needed for mokopuna to progress
- prompts and questions to connect with and recall previous learning
- asking mokopuna to make links to mathematics throughout their investigation in their own words
- asking the right mathematical questions to further their understanding of situations and to solve problems using mathematical ideas relevant to mokopuna
- providing visual prompts and resources to make connections to pāngarau concepts (e.g., using visual prompts of mathematical strategies, investigation process)
- trying a range of alternative representations
- using familiar contexts to consolidate concepts to more complex concepts
- supporting mokopuna to make connections to mathematical terminologies with stimuli and hanging them on classroom walls
- providing opportunities for activities and assessment tasks that allow kaiako and mokopuna to notice when mokopuna have mastered specific mathematical knowledge
- recognising what mokopuna will need to know to enable learning to be seen and heard as it occurs and is repeated
- sharing mokopuna progress, strategies for advancing projects and project outcomes in the kura with interested groups in the community
- guiding and facilitating with deliberate and explicit teaching as appropriate.

Grouping mokopuna:

- use flexible groups, based on the purpose of learning for lessons and kaupapa mathematics learning, rather than fixed long-term grouping
- you may group mokopuna in several ways in the same lesson (e.g., working as a whole class to demonstrate and discuss, before moving into smaller groups to investigate a situation or solve a problem).

Teaching Strategies

Provide opportunities to apply knowledge	• Mokopuna learn through doing, talking, writing/drawing, and modelling. • Plan time for mokopuna to consolidate what they have learned by repeating the process or task demonstrated by the teacher, moving from familiar situations towards applying to unfamiliar situations. • Plan tasks for mokopuna to apply knowledge and develop proficiency with problem solving and reasoning. • Adapt the task (e.g., start with familiar knowledge and gradually increase the complexity). Ensure a high ceiling, low floor approach with many points of entry and exit in activities and lessons. • Use a wide range of contexts to practice mathematical knowledge with so that mokopuna are able to use mathematical knowledge and skills in more than one situation, or context.
Teach and ask mokopuna to keep records of their learning	• Support mokopuna to record, review, connect, and synthesise ideas and reasoning. • Ask mokopuna to explain and represent their ideas using words, symbols, pictures, diagrams, and their working. • Allow time for mokopuna to practise vocabulary by making a display at a table, designing an anchor chart, taking photos and labelling them, or recording in their books. • Support mokopuna to organise their ideas, record, solve problems, summarise and reflect on what they have learnt using words, symbols or other representations. • Support mokopuna to recognise that sometimes mathematics is the context that they are investigating. It does not always have to be tied to contexts, situations or issues that sit outside of mathematics.

Co-requisites NCEA Pāngarau

All learners need to achieve a 20-credit co-requisite specific to te reo matatini or literacy and pāngarau or numeracy skills to be awarded any level of NCEA. The co-requisite is a one-off requirement.

Here is a link an explanation of the co-requisites for NCEA:
https://ncea-live-3-storagestack-53q-assetstorages3bucket-2o21xte0r81u.s3.amazonaws.com/s3fs-public/2024-03/Improved%20NCEA%20Co-requisite%20standards_TeReMaori_WEB.pdf?VersionId=okaWbub8am6LMYzWVuKOkGsawbqY3djI

Here is a link to some resources https://www2.nzqa.govt.nz/ncea/subjects/select-subject/te-reo-matatini-me-te-pangarau/

He Rauemi Te Reo Pāngarau

Ngā Ara Tohu Te Reo Pāngarau [PDF, 3.8 MB]. This is a teacher's resource that contains a range of information to plan, teach and how the co-requiste pāngarau is assessed. It also provides a range of useful on-line links and templates teachers can use to support their teaching and learning and teaching programme

Ngā Whanaketanga 5a (A3) [PDF, 164 KB]. This spreadsheet separates the broad learning outcomes in the pre 2024 marautanga into content for year 9 and 10

Whakaatu ai ngā ākonga kei te paearu i ngā tukanga Pāngarau (A3) [PDF, 278 KB] This resource explains the criteria that help students achieve the learning outcomes of co-requiste pāngarau unit standard in te reo Māori and English.

Link https://ncea.education.govt.nz/mi/te-reo-matatini-me-te-pangarau/pangarau/te-ako

Tūārere 5
Years 11–13

Whenu			
HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Toi Mokopuna			
Mokopuna are flexible, interested, curious, collaborative, creative and reflective investigators, willing to test strategies to solve problems for the school, themselves, their whānau, hapū and iwi.	Mokopuna are agile thinkers like their ancestors. Pāngarau ways of thinking and doing have been passed to them and will be passed on to others.	Mokopuna are confident and articulate in their reasoning, explaining, and communicating mathematical ideas and concepts, and are interested in the role of mathematics and its value as a different type of language.	Mokopuna are persistent and reflective, monitoring and adjusting their own thinking and performance by contextualising and identifying patterns and connections, constructing mathematics and their worlds.
Tohu Ako: Phase 5: Years 11–13			
Whāinga			
Mokopuna learning focuses on carrying out project-based investigations and justifying the mathematical techniques selected for analysing and solving problems. Evaluating solutions and situation reports through mathematical knowledge.	**Mokopuna learning focuses on** understanding the global application of mathematics for the wellbeing of whānau, hapū and iwi; using mathematical outcomes as evidence to guide future options and directions.	**Mokopuna learning focuses on** using specialised mathematical language to communicate investigative findings into specific areas, explaining the evidence to both mathematical experts and other interested audiences.	**Mokopuna learning focuses on** evaluate mathematical models and representations to gain insight into conjectures, hypotheses, and generalisations.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 5: Years 11–13			
Kia Mataara			
By the end of year 11, mokopuna: • appropriately apply core mathematical knowledge to widely experienced contexts and problems that are relevant to them (e.g., financial literacy) • plan and carry out a statistical investigation, make valid inferences about populations, and understand the importance of sample size • calculate probabilities and use them to guide future actions • investigate mathematical problems, generalise the results and connect these to other known results.	**By the end of year 11, mokopuna:** • use mathematics to support the aims and aspirations of te ao Māori through the careful interpretation and application of mathematical knowledge and practices that abide by Māori values • select the contexts of greatest significance to them and use mathematics to support te ao Māori.	**By the end of year 11, mokopuna:** • express relationships between variables using formulae, diagrams, charts and graphs • correctly lay out a series of steps in a calculation using mathematical symbols and conventions • explain the significance of the results of a statistical investigation in relation to the population being studied • discuss the significance of probabilities and the implications for possible future decisions • evaluate statistical reports and critically discuss their validity.	**By the end of year 11, mokopuna:** • generalise problems using algebraic expressions and manipulations • apply procedures to evaluate formulae and perform calculations • calculate perimeters, areas, and volumes of 2D shapes and 3D solids using correct units and precisions according to the context • logically deduce results about quantities, variables, geometrical shapes, and solids (e.g., using proportional reasoning, known general results, trigonometry, and Pythagoras' Theorem) • calculate rates of change from data and the gradients of graphs • construct graphs to relate between variables (e.g., graphs of quadratic functions) • use co-ordinate planes to solve problems (e.g., locations of objects, distances of journeys and areas of shapes) • begin to consider future career opportunities that involve advanced study in mathematics.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 5: Years 11–13			
Kia Mataara			
By the end of year 12, mokopuna: • conduct statistical investigations involving decisions about appropriate sampling methods and estimations of population parameters • make informal inferences, predictions, interpolations and extrapolations about statistical investigations • investigate situations that involve chance using theoretical distributions (e.g., normal) and simulations or experiments • make mathematical conjectures and conduct investigations to develop proofs • combine different elements of mathematical knowledge and apply them to local, general, and specialised contexts.	**By the end of year 12, mokopuna:** • connect mathematical and Māori knowledge where appropriate so that both bodies of knowledge are represented • identify commonalities and differences between mathematics and mātauranga Māori.	**By the end of year 12, mokopuna:** • represent contexts, problems and situations using series, calculus, co-ordinate geometry, algebraic formulae, relationships and graphs • discuss the significance of results obtained from mathematical and statistical investigations • evaluate statistical reports, critically assess the methods used and critically discuss their validity and implications.	**By the end of year 12, mokopuna:** • apply co-ordinate geometry techniques to points, lines and shapes located on a co-ordinate plane • use geometric and arithmetic sequences • use additive and multiplicative number patterns • apply trigonometry (including sine and cosine rules) and Pythagoras' theorem in 2 and 3 dimensional situations • manipulate rational, exponential, and logarithmic algebraic expressions • form and use linear, quadratic, and simple trigonometric equations and pairs of simultaneous equations (one of which may be non-linear) • apply calculus techniques of differentiation and integration to polynomials • calculate probabilities using a variety of methods (e.g., two-way tables, tree diagrams and computer simulations) • understand the pivotal role mathematics plays in technological development and innovation.
By the end of year 13, mokopuna: • conduct statistical investigations using experimental design principles, surveys, linear regression, time series, confidence intervals and the central limit theorem • create valid mathematical models that represent situations using permutations and combinations, networks, simultaneous equations, calculus, and probability distributions.	**By the end of year 13, mokopuna:** • use mathematical models to analyse situations and produce predictions and statements relevant to the aims and aspirations of Māori communities and wider society • analyse the potential tensions and contradictions that exist between mathematical and Māori ways of seeing the world • mitigate these potential tensions by identifying commonalities and differences between mātauranga Māori and mathematics and deciding how they will interact.	**By the end of year 13, mokopuna:** • discuss the use of mathematical models, explaining how the model was formed and its validity in relation to the situation • discuss the significance of results obtained from mathematical models • evaluate models and critically assess their applicability, validity, and implications.	**By the end of year 13, mokopuna:** • understand the most important knowledge of this year level, that is: - geometry of conic sections - permutations and combinations - techniques of curve fitting - networks and paths - simultaneous equations involving three equations - calculus differentiation, integration techniques, and differential equations - numerical methods to solve equations - Poisson, binomial and normal distributions - independent, combined, and conditional events • evaluate the importance of mathematics in New Zealand from economic and historical perspectives • have a career pathway for the near and medium term in mind and know how mathematics is relevant to their pathway.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 5: Years 11–13			
Te Ngako o te Whāinga			
Mā te Kaiako			
Year 11 **Support mokopuna to:** • develop conceptual understanding and procedural competency in core year 11 mathematics content • interpret problems in familiar contexts so that mathematics can be applied • carry out calculations and use mathematical diagrams to find results that meet their own aims and those of their whānau, hapū and iwi (e.g., creating a budget to show that a project is financially viable).	**Year 11** **Support mokopuna to:** • use mathematics in projects that are important to their village, local communities, whānau, hapū and iwi (e.g., rejuvenating natural landscapes, producing healthy food sources, maintaining marae) • use mathematics to help analyse issues of national importance.	**Year 11** **Support mokopuna to:** • explain how and why they use certain calculations and diagrams and how the results connect meaningfully with the problem, situation or context being investigated.	**Year 11** **Support mokopuna to:** • review their calculations and diagrams and consider improvements in efficiency and accuracy • review and evaluate whether their work is fit for purpose in the context of the situation or problem being solved • evaluate the validity and usefulness of their results.
Year 12 **Support mokopuna to:** • use concepts, such as coordinate geometry and elementary calculus to: - analyse relevant problems, and situations in context - produce results and conclusions that are relevant to them, their whanau, hapū and iwi.	**Year 12** **Support mokopuna to:** • utilise more advanced mathematical concepts and techniques to interrogate issues that are relevant to them, their whānau, hapū and iwi • think critically about how more advanced mathematical concepts and techniques connect with mātauranga Māori (e.g., how traditional navigation compares with GPS systems).	**Year 12** **Support mokopuna to:** • use language, symbols and representations that combine different strands of mathematics (e.g., how calculus combines measurement concepts and language with algebraic manipulations and graphs).	**Year 12** **Support mokopuna to:** • integrate different strands of mathematics (e.g., coordinate geometry connects algebra and geometry).
Year 13 **Support mokopuna to:** • continually strengthen their mathematical knowledge and skills to analyse and solve complex problems • carefully and critically use the technologies of the 21st century (e.g., Artificial Intelligence) in the service of Māori aspirations.	**Year 13** **Support mokopuna to:** • acknowledge the aims and aspirations of their Māori communities and consider how mathematics may be used to achieve them • select the most useful aspects of mathematics for the resolution of situations that challenge te ao Māori • use mathematics to produce results, information, and insights that will be useful to hapū and iwi.	**Year 13** **Support mokopuna to:** • use the specialised words, expressions, signs, symbols, and representations of mathematics correctly • use mathematics to explain and justify their ideas and contentions to different people in different contexts • explain mathematical topics to unfamiliar audiences.	**Year 13** **Support mokopuna to:** • construct and use complex mathematical models.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 5: Years 11–13			
Te Ngako o te Whāinga			
Mā te Mokopuna			
I am learning to: • organise my ideas, utilise my skills, and provide articulate responses to the guidance of teachers • be committed to my own achievement at this level by identifying mathematical aims and objectives for chosen projects • acknowledge the status of both Māori knowledge and mathematics in my planned mathematical investigations.	**I am learning to:** • seek contemporary and traditional knowledge to find the best ways of using mathematics to support te ao Māori • carefully select and combine the types of mathematical knowledge used to solve problems confronting Māori communities • use mathematics to add value for other people and communities • use technologies of all kinds (e.g., AI, computers, internet) ethically in accordance with te ao Māori, by including a description of ethical practices.	**I am learning to:** • continually increase my mathematical vocabulary (e.g., language related to calculus and coordinate geometry) • use mathematical language confidently and effectively when communicating my findings to relevant stakeholders • explain my mathematical ideas in a variety of ways appropriate to context.	**I am learning to:** • use algebraic skills to solve problems of increasing sophistication with real relevance and benefit that I can justify • create and use graphs to analyse patterns inherent in the vital situations being studied • use statistical concepts and techniques to explore the extent of a certain phenomenon (e.g., the spread of COVID-19 around the world) • calculate probabilities of outcomes that are of real relevance and make predictions about what might eventuate • integrate different parts of mathematical knowledge in a model that produces useful and relevant knowledge for both iwi Māori and wider society.

HE TANGATA tūhura ki te ao	HE URI WHAKAHEKE ki te whai ao	HE PUNA KŌRERO o te reo pāngarau	HE ĀKONGA mauri oho
Tohu Ako: Phase 5: Years 11–13			
Te Roanga o te Kōrero			
Characteristics of Mokopuna Mokopuna will acknowledge their responsibility to apply mathematical knowledge and skills to the problems that confront te ao Māori and global society (e.g., the environment, the rejuvenation of species of plants and animals). Mokopuna are guardians of the environment and te ao Māori and can use mathematics to alleviate problems that arise from the over-exploitation of the natural world.	**Characteristics of Mokopuna** Mokopuna are mediators Mokopuna are mediators between mātauranga Māori and mathematics, maintaining both as important bodies of knowledge. Mokopuna centralise their identity as Māori people who also have a high degree of mathematical knowledge, understanding, and practical skill. Mokopuna can realise their potential and responsibilities, becoming contributing citizens by drawing on their mathematical knowledge and skills.	**Characteristics of Mokopuna** Mokopuna aim to develop their mathematical language alongside te reo Māori and English. This involves the artful interweaving of Māori and mathematical modes of discourse to create greater clarity of expression for their ideas and justification of propositions. Mokopuna use mathematical language to achieve practical benefits for individuals and groups. Mokopuna have the skills to create and manipulate a range of symbolic representations of quantities and to interpret other devices such as graphs, diagrams, and mathematical models by drawing on their foundational mathematical language.	**Characteristics of Mokopuna** Mokopuna see themselves as continual learners with limitless knowledge to draw from. They also recognise their potential to create and share new knowledge with the world. Mokopuna will organise their own pathways for themselves, recognising that mathematical knowledge is helpful for making discerning and critical decisions. Mokopuna will be secure in their own mana and in their use of mathematics. Mokopuna are ready to move into the wider world as independent people who base their work in all spheres on a Māori perspective that integrates many different forms of knowledge and aims to resolve challenges in te ao Māori and in wider society.
Affirming Identity Mokopuna are aware of how others work things out, what others want to know about, and how to relate appropriately during mathematical investigations as part of their culture of mathematical investigation as they work both independently and collaboratively. Mokopuna are creative with pāngarau, and critical thinkers using pāngarau.	**Affirming Identity** Mokopuna are confident with mathematics culture, language and tikanga, and can articulate which parts of the mathematical processes are based on tīpuna ways of thinking and when mathematics content knowledge is useful but does not supersede tīpuna knowledge and tikanga.	**Affirming Identity** Mokopuna confidence to communicate about their mathematical ideas and findings is evident as they use more sophisticated mathematical content knowledge, particularly if they have real purposeful reasons to share and communicate to wider audiences how mathematical findings are meaningful to the situations they have been investigating.	**Affirming Identity** Mokopuna relate to mathematics knowledge for its own sake, and for the ways that they can use mathematics in their everyday lives to make decisions based on mathematical findings and cultural knowledge.
Development of Learning From Year 11 to Year 13			
Year 11	**Year 11 to Year 12 Specialised Mathematics**	**Year 12**	**Year 13**
Mathematical knowledge and processes are used to solve relevant problems and resolve issues in contexts that have a close connection to mokopuna experiences and wider contexts that will be encountered during the year.	In relation to pāngarau knowledge, the perspective shifts from familiar contexts to the conceptual structures of pāngarau. These conceptual structures are connected to important features in society and everyday life. This is the beginning of specialisation in pāngarau.	A major development is how different strands of mathematics are combined to produce critical systems that generate knowledge which gives insight into phenomena both within and outside the immediate experiences of mokopuna.	The specialisation continues, and it is in this year that we can expect to see the emergence of the mokopuna as a mathematician. This involves the creation of mathematical models, the critical analysis of their use, and the predictions and claims that result from them.

Tohu Ako: Phase 5: Years 11–13

Number

Learning Strategies

Mokopuna are learning to:

- use of a variety of complex strategies to solve problems involving:
  - natural numbers
  - prime numbers
  - rational and irrational numbers
  - integers (positive and negative numbers)
  - reciprocals
- use a range of strategies to solve multiplication and division problems for integers (positive/negative), decimals, fractions, indices, and percentages
- use a wide range of strategies to solve problems involving whole numbers, decimals, fractions, percentages and integers
- solve proportional problems using multiplication (e.g., If 12 balls of wool make 18 scarves, how many balls of wool are needed to make 15 scarves?).

Key Understandings

Year 11
Mokopuna are learning to:

- recognise the properties of numbers and identify suitable situations for their use, including:
  - natural numbers
  - prime numbers
  - rational and irrational numbers
  - integers (positive and negative numbers)
  - reciprocals
- calculate with numbers in scientific notation
- use rounding and estimation to predict outcomes and to check the reasonableness of calculations
- perform operations on positive and negative rational numbers in any form (whole numbers, fractions, and decimals), including taking integer powers and roots.
- perform operations with percentages, including increasing or decreasing quantity using a single multiplier, expand exponents and integers
- use fractions, decimals, and percentages, choosing the appropriate form
- apply rates and ratios, including proportional and inverse proportional reasoning
- calculate compound interest over a fixed amount of time, compounding annually, quarterly, monthly, and daily.

The Language of Number

Mokopuna know the following words:

- exponent
- inverse proportion
- prime number
- rational number
- irrational number
- reciprocal
- scientific notation.

Tohu Ako: Phase 5: Years 11–13

Algebra

Learning Strategies

Mokopuna are learning to:

- use graphs, tables, and rules to describe linear relationships found in number and spatial patterns
- use graphing methods of relationships to solve simultaneous equation problems
- use expressions and graphing of relationships to solve problems
- discuss multiple graphs to show patterns and relationships with a specific variable.

Key Understandings

Equations, Inequalities and Expressions

Year 11
Mokopuna are learning to:

- find solutions that maximise or minimise a quantity while meeting the constraints of the situation by making lists, tables, and graphs and comparing values
- operate on numeric and algebraic expressions with integer exponents, applying exponent rules
- add, subtract, multiply, and divide algebraic fractions with numeric denominator
- simplify, expand, and factorise algebraic expressions
- create and solve linear equations and inequalities
- form and solve pairs of simultaneous linear equations in two variables and give geometric interpretations
- form and solve quadratic equations with leading coefficient:
  e.g., $x^2 + bx + c = 0$
- substitute into, rearrange, and simplify algebraic expressions or formula
- interpret, graph, and express linear functions in any form
- for quadratic and exponential functions, with a positive integer base:
  - make a table and graph from an equation
  - interpret key information and features of a graph.
- recognise the relationship between elements of linear and quadratic patterns, write the equation to represent the rule, and use the rule to make predictions.

The Language of Algebra

Mokopuna know the following words:

- factorise
- exchange rate
- quadratic equation
- exponential equation
- spatial pattern
- simplify
- linear relationship
- inverse relationship
- exponential relationship
- quadratic relationship
- exponential formula.

Tohu Ako: Phase 5: Years 11–13

Measurement and Geometry

Learning Strategies

Mokopuna are learning to:

- recognise the complexity of the attribute being measured and use correct units
- solve measurement problems by splitting shapes into component parts, applying formulae, using measurement ratios and trigonometric approaches appropriately
- describe and show direction using vectors
- use symmetry of angles to find concurrent and parallel lines, polygons and polyhedrons
- use geometrical transformations and Pythagoras to solve problems
- use geometric methods to solve problems
- analyse and apply relationships and slope graphs to solve problems
- investigate trigonometric methods related to a specific variable
- test multiple methods for finding the gradient of a specific variable
- explore multiple methods for finding the area under the curve of a specific variable.

Key Understandings

Measurement

Year 11
Mokopuna are learning to:

- measure at a level of precision appropriate to the task
- connect significant figures and limits of accuracy
- apply the relationships between units in the metric system, including the units for measuring different attributes and derived measures
- calculate surface area and volumes of 3D shapes, including prisms, pyramids, cones, and spheres, using formulae
- use trigonometric ratio and theorem of Pythagoras to solve problems in 2D and 3D situations
- link transformations in the XY graphs of lines and parabolas to changes in equations.

Geometry

Mokopuna are learning to:

- use a coordinate plane or map to show points in common and areas contained by two or more loci
- deduce and apply the angle properties related to circles
- recognise when shapes are similar and use proportional reasoning to find an unknown length
- use trigonometric ratios and Pythagoras theorem in two and three dimensions
- compare and apply single and multiple transformations
- analyse symmetrical patterns by the transformations used to create them
- scale a shape by a factor and find the perimeter, area, or volume of the scaled shape.

The Language of Measurement and Geometry

Mokopuna know the following words:

- trigonometric ratio
- pyramid
- cylinder
- single transformation
- multiple transformations
- point of intersection
- points in common
- loci
- scale
- trigonometric graph
- intersecting lines
- parallelogram
- polygon
- polyhedron
- symmetrical patterns
- derived measures.

Tohu Ako: Phase 5: Years 11–13

Measurement and Geometry

Patterns and Relationships

Year 11 to Year 12 Specialised Mathematics

- Measurement knowledge and geometry knowledge have been integrated into Algebra and Calculus.
- Calculus is a significant extension of knowledge that allows for the expression of all types of motion, involving different variables, not just the movement of physical objects like a car, airplane, ball, or person.

Year 12 Mokopuna are learning to:	Year 13 Mokopuna are learning to:
• use mathematical modelling to approximate applied situations.	• use mathematical modelling to approximate applied situations.
• manipulate algebraic expressions, including algebraic fractions, by: - simplifying, rearranging, expanding, and factorising - adding, subtracting, multiplying, and dividing - raising to integer and unit fraction exponents: e.g., $2^3 = 2 \times 2 \times 2 = 8$.	• manipulate algebraic expressions, including those involving roots, by: - simplifying, rearranging, expanding, and factorising - adding, subtracting, multiplying, and dividing - rationalising the denominator - raising to rational exponents.
• form, solve, and graph quadratic functions, equations, and inequalities with rational coefficients, including completing the square and using the discriminant.	• form, solve, and graph polynomial functions, equations, and inequalities with rational coefficients, including those with complex roots.
• write, graph, and solve simultaneous equations.	• create and use systems of simultaneous equations, including three linear equations and three variables, and interpret the solutions in context.
• investigate linear and non-linear graphs and relate the characteristics of the graph and the relation being modelled.	• create and interpret graphs of functions, inverse functions, and reciprocal functions.
• relate graphs and their gradients • use differentiation and anti-differentiation of expressions involving polynomials • use coordinate geometry involving points, straight lines, and curves.	• use the geometry of conic sections • manipulate complex numbers and present them graphically • use curve fitting, logarithms and linear programming to model situations • choose and apply a variety of differentiation, integration, and anti-differentiation techniques to functions and relations, using both analytical and numerical methods • form differential equations and interpret the solutions.
• derive and use sine rule, cosine rule, and area formula (e.g., ½ ab sin C) to find triangle sides, angles, and area.	• use trigonometric identities to simplify and rearrange trigonometric expressions.
• use network methods to find optimal routes between one place and another.	• create network diagrams to find optimal solutions and critical paths.

The Language of Measurement and Geometry

Mokopuna know the following words:	Mokopuna know the following words:
• network • sum of arithmetic series • sum of geometric series • point • straight line • curve • fractional algebraic expression • exponential • inverse exponential • route • trigonometric • differentiation • anti-differentiation • polynomials • sine rule • cosine rule • area formula • fractional exponent.	• composite number • system of simultaneous equations • non-liner equation • trigonometric equation • polynomial equation • non-linear equation • conic section • differentiation • anti-differentiation • integrate • linear programming • curve fitting • logarithm • trigonometric identities • network diagrams.

Tohu Ako: Phase 5: Years 11–13	
Measurement and Geometry	
Calculus	
Year 12 **Mokopuna are learning to:**	**Year 13** **Mokopuna are learning to:**
• relate graphs and their gradients • use differentiation and anti-differentiation of expressions involving polynomials.	• identify discontinuities and limits of functions • choose and apply a variety of differentiation, integration, and anti-differentiation techniques to functions and relations, using both analytical and numerical methods • form differential equations and interpret the solutions.
The Language of Calculus	
Mokopuna know the following words:	**Mokopuna know the following words:**
• differentiation • anti-differentiation • polynomial.	• discontinuities • limits of functions • differential equation • integration • anti-differentiation • analytical methods • numerical methods.

Tohu Ako: Phase 5: Years 11–13

Statistics and Probability

Learning Strategies

Mokopuna are learning to:

- examine the process and results of others' statistical investigations to evaluate whether their claims are believable and reasonable
- understand and explain why statistical investigations might be presented in particular ways
- investigate chance situations by making connections between experimental results and theoretical models
- pose investigative questions about a wider population
- collect samples of multivariate data and analyse these using displays to find patterns within, between, and beyond the data, and to notice unusual values
- recognise whether data needs to be cleaned and discuss appropriate reasons for variations in the data
- communicate their findings with appropriate displays and generalise in context
- explore multivariate, bivariate, and time series data and apply methods of data sovereignty
- evaluate statistical investigations based on sampling techniques and use statistical methods to make a formal inference of two groups
- evaluate the predictions that arise from time series and bivariate data investigations
- use probability techniques, probability concepts, and distribution methods to make a prediction.

Key Understandings

The Problem

Year 11
Mokopuna are learning to:

- investigate multivariate datasets for observational studies and investigate paired comparisons for experiments by:
  - posing summary and comparison investigative questions about populations
  - posing investigative questions for time-series and relationships, and for experiments
  - making predictions or assertions about expected findings.

Planning

Mokopuna are learning to:

- plan how to collect or source data to answer investigative questions for observational studies and experiments, including:
  - identifying and justifying the variables needed to answer the investigative question
  - planning and justifying how to make valid and reliable measures for the variables (when collecting) or interrogating sourced datasets to know what the variables measure and how they were measured
  - determining if the investigative question is about a group or a population, if it is a population, a sample from the population is taken to answer the investigative question
  - determining the sample size needed for categorical and numerical data in order to make a call about what is happening in the population
  - identifying the group or population of interest or "who" the data was collected from
  - using a set of interrogative questions that check ethical practices that should be considered throughout the entire statistical enquiry cycle.

Data Collection

Mokopuna are learning to:

- use random samples to collect or source data about a population for summary and comparison situations
- create a data dictionary (collected data) or find the metadata (sourced data)
- store data and ensure data is safe and confidential
- make decisions about the validity of data and make simple edits and clean data if appropriate
- carry out the data collecting process
- sort data in preparation for analysis.

Analysis

Mokopuna are learning to:

- create multiple visualisations and see the connections between these visualisations
- recategorise or construct new variables if needed
- identify and communicate features in context (trends, relationships between variables, and differences within and between distributions) using multiple displays
- produce formal inferences based on the data about the population
- justify findings using displays and measures and evaluate other.

Tohu Ako: Phase 5: Years 11–13

Statistics and Probability

Ethical Considerations - Data Sovereignty

Year 11
Mokopuna are learning to:

- follow appropriate ethical conventions such as informed consent of participants in a survey and maintaining anonymity of participants
- abide by additional indigenous ethical considerations and the issue of data sovereignty.

Conclusion

Mokopuna are learning to:

- use evidence from analysis
- generalise beyond the sample to the population, providing evidence and accounting for uncertainty (summary and comparison situations)
- provide explanations for observed patterns in the data (time series and relationship situations)
- reflect on predictions or assertions
- evaluate predictions or assertions
- critically evaluate their investigation for the different phases of the statistical enquiry, with an awareness of variability
- structure their evidence and findings into a coherent whole that communicates the entire statistical enquiry to a non-specialist
- evaluate statistical reports in the media by relating the displays, statistics, processes, and probabilities used to the claims made.

Probability

Mokopuna are learning to:

- plan and conduct probability experiments for chance-based situations, including undertaking many trials using technology, by:
  - posing investigative questions
  - identifying outcomes for the investigative question posed and anticipating what might happen
  - identifying model assumptions and limitations for the chance-based situation
  - constructing a model using theoretical probabilities, probabilities based on past data, or conjectured probabilities
  - deciding the number of trials, tools to be used, and recording method
  - running simulations and recording data
  - creating data visualisations for the distribution of observed outcomes from probability experiments and the distribution of possible outcomes for theoretical probability models
  - describing what these visualisations show
  - finding the probability estimates for the different outcomes
  - identifying similarities and differences between their findings and those of others
  - reflecting on anticipated outcomes
  - identifying, and giving potential reasons for, similarities and differences between their findings from probability experiments and associated theoretical probabilities.

The Language of Statistics and Probability

Mokopuna know the following words:

- population
- statistical enquiry
- summary question
- comparison question
- data sovereignty
- indigenous ethics
- multivariate data
- formal inference
- inference
- conjectured probabilities
- meta data
- model assumptions
- theoretical probability
- random sample
- experimental inference
- time series data
- bivariate data.

Tohu Ako: Phase 5: Years 11–13

Statistics and Probability

Year 11 to Year 12 Specialised Mathematics

- Statistics and Probability are distinct strands in Year 12 and 13.
- The significant extension and deepening is the focus on distributions as the foundation of mathematical models.

Statistical Analysis

Year 12 Mokopuna are learning to:	Year 13 Mokopuna are learning to:
• implement the statistical inquiry cycle: - construct structured-data from sources such as text, images, sounds, movements (including spatio-temporal), experiences (e.g., opinions, feelings) - identify times and outcomes for learning about statistical features and exploring data - conduct surveys using random sampling, conduct experiments and use existing data sets - evaluate the choice of measures for variables and the sampling and data collection methods used - use relevant contextual knowledge, exploratory data analysis, and statistical inference - follow the conventional ethical considerations for statistical investigations - relate indigenous ethics to extend conventional ethical considerations - engage with the issues of data sovereignty. • generate inferences from statistical surveys and experiments: - make informal predictions, interpolations, and extrapolations - use sample statistics to make point estimates of population parameters - recognise the effect of sample size on the variability of an estimate. • evaluate statistically based reports: - interpret risk and relative risk - identify sampling and possible non-sampling errors in surveys, including polls.	• implement the statistical inquiry cycle: - manipulate and restructure data from sources such as text, images, sounds, movements (including spatio-temporal), experiences (e.g., opinions, feelings) - identify times and outcomes for learning about statistical features and exploring data - conduct surveys, conduct experiments using experimental design principles, and use existing data sets - find, use, and assess appropriate models (including linear regression for bivariate data and additive models for time-series data), seek explanations, and make predictions - use informed contextual knowledge, exploratory data analysis, and statistical inference - relate conventional and indigenous ethics to whakapapa, data sovereignty and protection of indigenous taonga - explain the findings and evaluate all aspects of the investigation. • generate inferences from statistical surveys and experiments : - determine estimates and confidence intervals for means, proportions, and differences, recognising the relevance of the central limit theorem - use methods such as resampling or randomisation to assess the strength of evidence - evaluate a wide range of statistically based reports, including surveys and polls, experiments, and observational studies - critique causal-relationship claims - interpret margins of error.

The Language of Statistics

Mokopuna know the following words:	Mokopuna know the following words:
• random sampling • experiment • sampling error • non-sampling error • population parameter • data sovereignty • relative risk • interpolations • extrapolations.	• margin of error • media report • statistical survey • experiment • data sovereignty • central limit theorem • observational studies • causal-relationship claims.

Tohu Ako: Phase 5: Years 11–13

Statistics and Probability

Probability

Year 12 Mokopuna are learning to:	Year 13 Mokopuna are learning to:
• investigate situations that involve elements of chance: - compare theoretical continuous distributions, such as the normal distribution, with experimental distributions - calculate probabilities, using tools such as two-way tables, tree diagrams, simulations, and technology - explore changing the features of probability and probability distribution models and discuss their effects - interpret and critique data-based, chance-based, and model-based information and practices, data visualisations, and claims from a variety of sources including media.	• investigate situations that involve elements of chance: - calculate probabilities of independent, combined, and conditional events - calculate and interpret expected values and standard deviations of discrete random variables - apply distributions such as the Poisson, binomial, and normal.

The Language of Statistics and Probability

Mokopuna know the following words:	Mokopuna know the following words:
• theoretical distribution • normal distribution • experimental distribution • two-way tables • tree diagrams • simulations • probability distribution models • media.	• element of chance • Poisson distribution • binomial distribution • normal distribution • discrete random variable • independent events • dependent events • conditional events.

Hei Tautoko i te Ako

Pedagogical Guidance

Ensure the learning is accessible to all mokopuna

Kaiako are teachers of mokopuna, they are teachers of language, and they are teachers of learning.

Design learning for all mokopuna by:

- providing opportunities to solve problems (e.g., ask critical questions, plan activities that prompt discussion, evaluation and assessment of critical issues relevant to mokopuna)
- using prompts (e.g., mathematical terms, previous learned concepts, diagrams, and materials, including use of digital tools)
- adapting the task (e.g., applying mathematical strategies and concepts into context, and gradually increase the complexity)
- asking questions to redirect thinking or encourage connections
- chunking learning into smaller parts
- presenting ideas using different representations or materials
- providing opportunities to present findings from mathematical investigations to a wider audience
- further explicit teaching (reteaching), demonstrating, and consolidating
- changing the context to one that is more familiar to the mokopuna.

Support mokopuna with scaffolds such as:

- tailored teaching of specific mathematical skills needed for mokopuna to progress
- prompts and questions to connect with and recall previous learning
- asking mokopuna to make links to mathematics throughout their investigation in their own words
- asking the right mathematical questions to further their understanding of situations, to solve problems using mathematical ideas relevant to mokopuna
- providing visual prompts and resources to make connections to pāngarau concepts (e.g., using visual prompts of mathematical strategies, investigation process)
- trying a range of alternative representations
- using familiar contexts to consolidate concepts to more complex concepts
- supporting mokopuna to make connections to mathematical terminologies with stimuli and hanging them on classroom walls
- providing opportunities for activities and assessment tasks that allow kaiako and mokopuna to notice when mokopuna have mastered specific mathematical knowledge
- recognising what mokopuna will need to know to enable learning to be seen and heard as it occurs and is repeated
- sharing mokopuna progress, strategies for advancing projects and project outcomes in the kura with interested groups in the community
- guiding and facilitating with deliberate and explicit teaching as appropriate.

Grouping mokopuna:

- use flexible groups, based on the purpose of learning for lessons and kaupapa mathematics learning, rather than fixed long-term grouping.
- you may group mokopuna in several ways in the same lesson (e.g., working as a whole class to demonstrate and discuss, before moving into smaller groups to investigate a situation or solve a problem, split into other groups, mokopuna choose the groups that are most useful for their focus).

Teaching Strategies

Provide opportunities to apply knowledge	• Mokopuna learn through doing, talking, writing/drawing, and modelling. • Plan time for mokopuna to consolidate what they have learned by repeating the process or task demonstrated by the teacher, moving from familiar situations towards applying to unfamiliar situations. • Plan tasks for mokopuna to apply knowledge and develop proficiency with problem solving and reasoning. • adapting the task (e.g., start with familiar knowledge and gradually increase the complexity) Ensure a high ceiling, low floor approach with many points of entry and exit in activities and lessons. • Use a wide range of contexts to practice mathematical knowledge with so that mokopuna are able to use mathematical knowledge and skills in more than one situation, or context.
Teach and ask mokopuna to keep records of their learning	• Support mokopuna to record, review, connect, and synthesise ideas and reasoning. • Ask mokopuna to explain and represent their ideas using words, symbols, pictures, diagrams, and their working. • Allow time for mokopuna to practise vocabulary by making a display at a table, designing an anchor chart, taking photos and labelling them, or recording in their books. • Support mokopuna to organise their ideas, record, solve problems, summarise and reflect on what they have learnt using words, symbols or other representations. • Support mokopuna to recognise that sometimes mathematics is the context in itself that they are investigating, it does not always have to be tied to outside of maths contexts, situations, or issues all the time.

NCEA Pāngarau

Link to NCEA Pāngarau https://www2.nzqa.govt.nz/ncea/subjects/select-subject/pangarau/

This is a link to the administrative page for NCEA levels 1-3 for pāngarau in either te reo Māori or English. This page contains resources and tools to help support the planning, teaching and learning associated with internal and external NCEA assessments:

- the assessments dates Ngā rā aromatawai 2024
- the standards: Ngā paerewa paetae
- assessment exemplars, assessment schedules, assessment reports Ngā aromatawai o mua, ngā tauaromahi, ngā mahere aromatawai me ngā pūrongo aromatawai
- specifications, assessment tasks and assessment plan: Ngā tautuhinga, ngā tūmahi, me ngā mahere aromatawai
- supporting documents for the moderator: Ngā rauemi tautoko o te kaiaromatai mō ngā aromatawai ā-roto
- other resources and information: Ētahi atu rauemi, taipitopito kōrero hoki

Pāngarau Kuputaka

A

ahu-3	3-dimensional
ahu-2	2-dimensional
āhuatanga pūmau	invariant property
aromātai	to evaluate
aroturuki	monitor

H

hātepe	steps; algorithm
hononga	relationship
hōpara	explore
horopaki	context
huritao	reflect upon

K

kīanga	phrase; expression
kīanga taurangi	algebraic expression
kitenga	finding
kōwhiringa	option; choice

M

māramatanga	understanding
matike	arise

O

otinga	outcome; effect

P

pākiki	curious
pānga rārangi	linear relationship
panoni	transformation; change
pāpātanga	rate
pāpono	event
parahau	justify
pūāhua	situation (like context)
pūhui	compound
pūkenga	skill
pūnaha	system
pūtahitanga	institution
pūtake	purpose
putanga, putanga iho	outcome

R

rahinga	quantity
rārangi whakarara	parallel lines
raraunga houanga	time series data
raraunga matarua	bi-variate data
raraunga motumotu	discrete data
raraunga whakarōpū	category data
raumata	net (of a solid shape)
rautaki	strategy
rawa	natural resources; resource; materials

T

takahuri	amend; alter; change
takirua raupapa	ordered pair

tāpaetanga kōrero	assertion
tāpua	significant
taputapu	tool; instrument
tau hiato	complex number
tau toitū	prime number
tau tōpū	integer
tauira raupapa	sequential pattern
tauira tāruarua	repeating pattern
taunaki (-tanga)	evidence
taupori	population
taupū	exponent
taurangi	variable; algebra
taurea	multiple
taurite	equilibrium
tautohe(tia)	contested
tautohu	identify
tauwehe	factor
tāwariwari	flexible
tipu	plant
tirohanga	perspective
tō	pull
tohu	symbol
toitū	lasting; sustainable
torotoro	explore
tūāhua	description of a person or thing
tuakiri	identity
tūāpapa	foundation; basis
tūārere	phase
tuhi	record
tūhuratanga tauanga	statistical investigation
tukutuku	grid

U

uara	value
urutau	adapt

W

wahapū	articulate
waihanga	shape; devise; create
wāwāhi	partition
whai wāhi (ki) ...	participate; have opportunity
whakaahua	describe
whakaahuahanga	representation
whakaari raraunga	data display
whakaata	reflect
whakaawhiwhi	round (a number)
whakamahi	use; apply
whakamātau tika	fair test
whakaoti rapanga	problem-solving
whakatakoto	outline; propose
whakataurite	compare; contrast
whakatinana	implement
whakatutuki (matea tangata)	meet (peoples' needs)
whakauru	engage; insert; include
whakaute	respectful
wheako	experience