Te Poutāhū
Curriculum Centre

Te Mātaiaho

The New Zealand Curriculum

MATHEMATICS AND STATISTICS YEARS 9–13

Mātai aho tāhūnui,
Mātai aho tāhūroa,
Hei takapau wānanga
E hora nei.

Lay the kaupapa down
And sustain it,
The learning here
Laid out before us.

Te Tāhuhu o te Mātauranga
Ministry of Education

Te Kāwanatanga o Aotearoa
New Zealand Government

DRAFT

Foreword

Tēnā koutou katoa,

It is my pleasure to introduce the revised mathematics and statistics learning area for years 9–13 of the New Zealand Curriculum. This marks the first time we have developed a national curriculum that is knowledge-rich, year-by-year, sequenced, and based on the science of learning. During the quality assurance process, the Education Review Office engaged with experts from around the world to give us confidence that our maths curriculum is internationally comparable and sets our students up for success. For years 11–13, the learning area is also broken out into separate subjects (mathematics and statistics) to more clearly align with NCEA achievement standards and senior secondary programmes of learning.

This new curriculum ensures clarity about what students need to understand, know, and do each year, while also providing teachers with a clear framework, resources, and supports to guide their classroom practice.

Our aim is to reduce ambiguity, empowering educators to teach with confidence and ensuring that all students have access to a high-quality education that prepares them for success in the future. This curriculum reflects the dedication of many individuals and groups working together to offer our young people the best educational opportunities, including teachers, principals and educational leaders from around the country.

In the mathematics and statistics learning area, students will develop a deep understanding of symbolic representation, reasoning, and abstraction. They will explore and explain patterns and relationships in quantity, space, time, data, and uncertainty. It is my ambition that every student in New Zealand will not only engage with mathematics and statistics but master the knowledge and skills necessary to excel, developing confidence and capability as mathematicians and a sense of personal curiosity and enjoyment that comes from making sense of the world around them. This new curriculum, alongside the years 0–8 content released last year, will form a comprehensive framework that ensures a strong, consistent foundation for mathematics education across Aotearoa New Zealand and enables us to invest in consistent, high-quality resources and professional development.

Ultimately, this new content is about raising achievement and closing the equity gap. Every learner should have the opportunity to realise their full potential and thrive in future study, work, society, and life.

I encourage you to engage with this learning area content and provide your feedback so that we can ensure your expertise as classroom teachers and experts in mathematics education is reflected in the final version of this learning area, due for release later this year. Every piece of feedback is incredibly valuable, and I look forward to working together to ensure that this curriculum is truly world-leading and sets up both our teachers and students for success.

Ngā mihi nui

Hon Erica Stanford (Minister of Education)

Contents

The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The New Zealand curriculum is a knowledge-rich curriculum that prioritises and explicitly describes the teaching that must be taught each year. It is deliberately sequenced to enable students to build knowledge, skills, and competencies systematically over time. It supports teachers to design teaching and learning programmes that bring it to life in the classroom, through local, national, and global contexts.

The curriculum builds on understandings of how we learn from the science of learning to identify five key themes:

- **We learn best when we experience a sense of belonging in our learning environments and feel valued and supported.**

  Students bring with them different cultural identities, knowledge, belief systems, and experiences. They need to see that these are valued and reflected in a school environment characterised by strong relationships and mutual respect between teachers and students and sensitivity towards students' individual needs, emotions, cultures, and beliefs.

- **A new idea or concept is always interpreted through, and learned in association with, existing knowledge.**

  The amount of existing knowledge a student has about a topic and the degree to which they are able to connect it to the topic influence both the quality and ease of learning. Recognising and drawing on students' prior knowledge are connected to improved learning outcomes.

- **Establishing knowledge in a well-organised way in long-term memory reduces cognitive load and allows the knowledge to be applied and transferred.**

  Building a knowledge base requires active engagement and multiple opportunities to learn new knowledge and connect it to existing knowledge structures. When a learner encounters a new situation, if they have the appropriate knowledge in the form of well-structured schema, they are more likely to be able to apply this existing knowledge to the new situation. If they do not have the appropriate knowledge in long-term memory, they will be dependent on their more limited working memory and may find the new information causes cognitive overload.

- **Our social and emotional wellbeing directly impacts on our ability to engage in learning.**

  Positive social wellbeing frees up cognitive capacity, enhances the process of learning new knowledge and skills, and leads to deeper, more permanent learning for students. Conversely, stress, anxiety, and negative emotions inhibit a student's ability to learn. For each student, there will be different factors that impact, positively or negatively, on their social and emotional wellbeing. The influence of these factors is dynamic, varying over time and even within the course of a day.

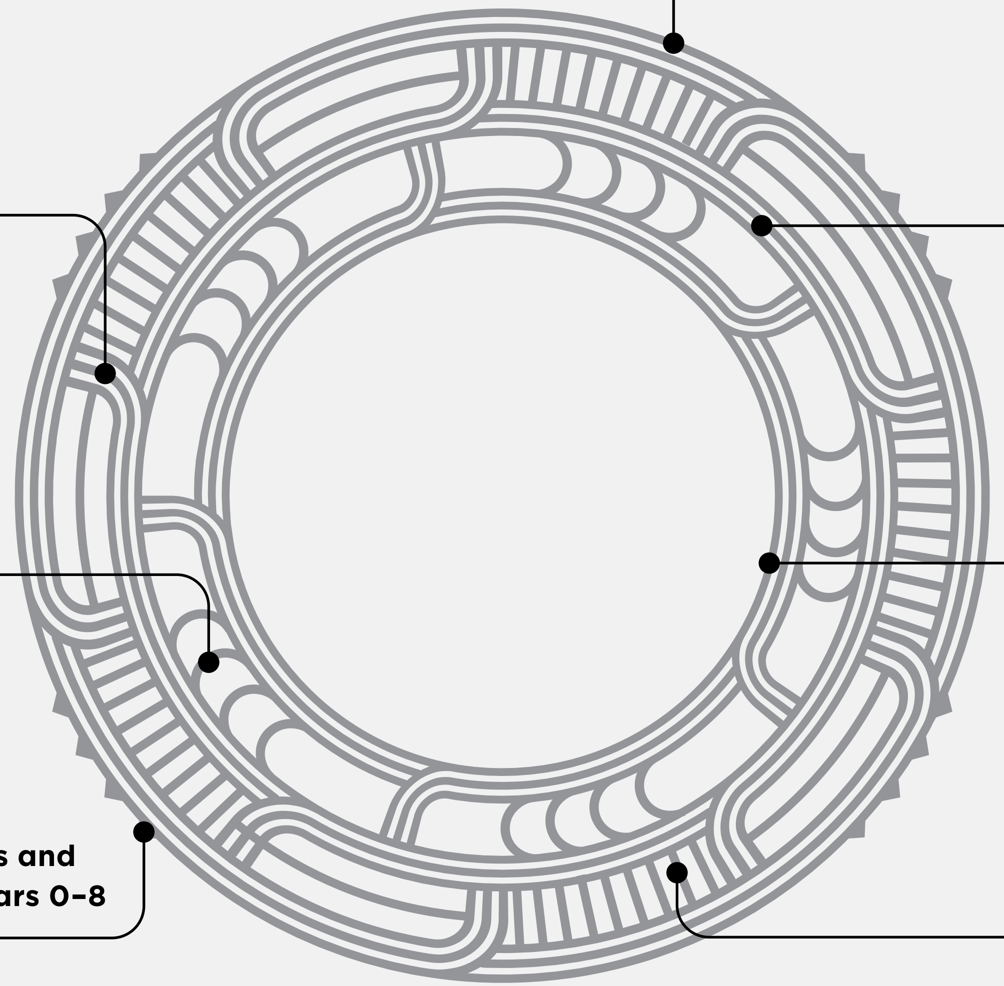
- **Motivation is critical for wellbeing and engagement in learning.**

  Motivation develops when students feel that three basic needs are met – autonomy, a sense of competence, and connection. Success in learning helps to build motivation.

The whakapapa of Te Mātaiaho draws together these themes in a structure that is coherent and inclusive of all students.

The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The design of this framework encompasses seven curriculum components. Te Mātaiaho as a whole weaves together these components, all of which begin with the word 'mātai', meaning to observe, examine, and deliberately consider.



Mātairangi | The guiding kaupapa

The overarching kaupapa guiding the curriculum, based on the science of learning and ensuring excellent and equitable outcomes for students

Mātai ki te rangi, homai te kauhau wānanga ki uta, ka whiti he ora. | Look beyond the horizon, and draw near the bodies of knowledge that will take us into the future.

The outer rings represent our guiding kaupapa.

Mātaiahikā | Relationships with tangata whenua and local community

Learning through relationships with tangata whenua and local communities

Mātai kōrero ahiahi. | Keep the hearth occupied, maintain the stories by firelight.

Poutama curves represent relationships with tangata whenua and the community.

Mātainuku | Creating a foundation

The curriculum principles (e.g., holding high expectations, and enabling all students to access the full scope of the curriculum)

Mātai ki te whenua, ka tiritiria, ka poupoua. | Ground and nurture the learning.

The centre rings represent the foundation and calls to action.

Mātaioho | National curriculum – contextualised

The process by which schools bring the national curriculum to life through local, national, and global contexts

Mātai oho, mātai ara, whītiki, whakatika. | Awaken, arise, and prepare for action.

Unaunahi scales represent wealth of knowledge, purpose, and know-how.

Mātaitipu | Vision of young people

The educational vision of young people, as conceived by young people

Mātaitipu hei papa whenuakura. | Grow and nourish a thriving community.

The inner rings and circular space represent the vision and students at the centre.

Mātaiaho | Learning areas

The eight learning areas, which each include a purpose, big ideas, knowledge, and practices, year-by-year

Mātai rangaranga te aho tū, te aho pae. | Weave the learning strands together.

Taratara-a-kae niho notches represent diversity, resilience, and mana.

› **Mathematics and statistics years 0–8**

Mātairea | Supporting progress

The whole schooling pathway and the overarching focus for year-by-year learning and progress

Mātai ka rea, ka pihi hei māhuri. | Build and support progress.

Niho kurī lines represent building and supporting the development of students.

Learning areas

The curriculum has eight learning areas: English, the arts, health and physical education, learning languages, mathematics and statistics, science, social sciences, and technology. Together they provide the basis for a broad, general education for the first four phases of learning (years 0–10) and collectively lay a foundation for specialisation in phase 5 (years 11–13).

Each learning area is knowledge-rich. This knowledge has been carefully chosen to support all students in their schooling pathway and is framed using Understand, Know, and Do:

- **Understand** – the deep and enduring big ideas and themes that students develop understanding of over the phases
- **Know** – the meaningful and important concepts and topics at each phase that enrich students' understanding of the big ideas and themes and that students study using the practices
- **Do** – the practices (skills, strategies, and processes) that bring rigour to learning and support the development of the key competencies.

A **progression model** provides the structure that sequences the knowledge. It supports all students to develop greater:

- breadth and depth of knowledge and understanding, through engaging with increasingly complex and ambiguous contexts
- refinement and sophistication in their use of competencies, practices, strategies, processes, and skills
- ability to connect, transfer, and apply new learning in meaningful contexts
- knowledge and awareness of themselves as learners
- effectiveness when working with others.

Content of the learning areas

Knowledge and progression are reflected in how the learning areas are organised.

Purpose statement and UKD overview

A purpose statement for each learning area describes the learning area's contribution to the lives of students. It is followed by an overview of Understand, Know, and Do, which gives a broad view of the big ideas, themes, concepts, topics, and practices that underpin the learning area.

Teachers use the purpose statement and UKD overview to develop an understanding of the learning area, so that they can share its benefits with students.

Learning area structure

For each learning area, this section outlines its structure and the changes it undergoes over five phases of learning, particularly in the final phase, where students may opt to specialise and choose from a range of subjects.

There are five phases of learning, spanning years 0–13. Each phase covers two to three years of schooling, which reflects how most schools organise learning across year levels.

A **critical focus** for each phase establishes a sustained, strengths-based, holistic focus on the student and their social, emotional, and cognitive learning at this stage of their schooling journey. Each critical focus builds on the phase before and is reflected in the content of the learning area for the phase.

The critical focuses are:

- **Phase 1** (years 0–3): Thriving in environments rich in literacy and maths
- **Phase 2** (years 4–6): Expanding horizons of knowledge, and collaboration
- **Phase 3** (years 7–8): Seeing ourselves in the wider world and advocating with and for others
- **Phase 4** (years 9 -10): Having a purpose and being empathetic and resilient
- **Phase 5** (years 11 – 1)3: Navigating pathways and developing agency to help shape the future

Teachers use the critical focus of each phase in their selection and design of topics and activities.

Teaching guidance

Each learning area also draws from the science of learning and wider education theory to provide a knowledge base and guidance for teachers. Teachers use this to help them make purposeful decisions about how to teach the learning area's content in ways that are inclusive of all students.

The guidance is organised under three headings:

› Designing a comprehensive teaching and learning programme
› Using assessment to inform teaching
› Planning

Progress outcomes

In each learning area, there is one progress outcome for each phase.

The progress outcomes outline expectations. They act as signposts at the end of each phase of learning and indicate what students should sufficiently understand, know, and be able to do at key points in the schooling pathway.

The content of each progress outcome is organised using the Understand–Know–Do framework and reflects the critical focus of the phase. While the Understand statements remain the same over the five phases, students' depth of understanding increases as their knowledge of the learning area's content grows and their use of the practices of Do develops.

The progress outcomes help teachers maintain an overview of what students learnt in the prior phase and the expectations of achievement for the next phase, and so are key for planning, along with the more detailed teaching sequences (described below).

Teachers should also use the progress outcomes to take an 'on-balance view' of student progress and achievement. Schools can use information from twice-yearly, standardised assessment tools to help develop an on-balance view.

The following statements describe this view:

› Are students **using learning from the progress outcome of the previous phase to make sense of new learning in the current phase?** This demonstrates how well they can connect new learning to what they already know. It generally occurs in the first year of a phase.
› Are students **consolidating the learning expressed in the progress outcome** in a wide range of contexts? This demonstrates how well and confidently they are using their new learning. This generally occurs in the second year of the phase.
› Are students **secure in the learning described in the progress outcome within an increasingly complex range of contexts?** Are they showing greater depth of knowledge, understanding, and application as they use their new learning and prepare for the challenges of the next phase? This generally occurs towards the end of the final year of the phase.

On-balance views enable teachers and leaders to identify the learning to prioritise next. They can also be used to report to parents.

Teaching sequences

Each phase has a year-by-year teaching sequence. These sequences support teachers to know what to teach and when and how to teach it as students work towards the progress outcome for the phase. They have been organised to support students to revisit ideas, knowledge, and practices in ways that deepen their learning and enable them to use it at the next phase.

There are two parts in a teaching sequence: statements of **what** to teach, and 'teaching considerations' for **how** to teach:

› the 'what to teach' statements are preceded by the stem 'Informed by prior learning …', which reminds teachers to use their professional judgement and assessment information when selecting what content to teach
› the teaching considerations help teachers to know 'how to teach' this content in response to students' prior knowledge, strengths, and experiences.

The teaching sequence tables should be viewed both vertically and horizontally. Looking down the columns helps teachers know what to plan for in a year's programme. Looking across the rows at the statements for the same concept in the preceding and following years helps teachers to recognise prior learning that students may come with and to consider how they might extend this year's learning. It also helps teachers in taking an on-balance view of their students' progress (see above), and it is a strong support when planning for mixed level classes.

The approach of the year-by-year teaching sequences changes in phase 5, as the content become more discipline-focused.

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Te Mātaiaho

The New Zealand Curriculum

MATHEMATICS AND STATISTICS YEARS 9–13

Purpose statement

Ānō me he whare pūngāwerewere.
Behold, it is like the web of a spider.

This whakataukī celebrates intricacy, complexity, interconnectedness, and strength. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

In the mathematics and statistics learning area, students learn about and appreciate the power of symbolic representation, reasoning, and abstraction. They learn to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. As they achieve deep conceptual understanding and procedural fluency in the learning area, students can accurately and efficiently use mathematics and statistics as a foundation for new learning and to solve problems.

Students engage with mathematics and statistics through the exploration of problems, patterns, and trends and appreciate the everyday value of this learning in many areas of their lives, such as personal finance, health, dance, and design. Every student in New Zealand can engage in mathematics and statistics and discover personal enjoyment and curiosity in their learning.

Throughout their learning, students engage with diverse perspectives as they apply their mathematical and statistical understandings. They also learn that mathematics and statistics has an evolving history; many cultures have contributed to, and continue to contribute to, innovations that shape our current thinking.

As they move through the phases of the learning area, students come to understand the value of mathematical and statistical investigation as a lens for collective local, national, and global challenges. Mathematics and statistics allow us to engage with important societal matters, such as the robust and ethical gathering, interpretation, and communication of data, and the use of valid and reliable data to challenge misinformation and disinformation.

Learning in mathematics and statistics builds literacy by developing students' skills in oral and written communication, reasoning, and comprehension. The learning area opens pathways into a wide range of industries that rely on mathematical and statistical knowledge and reasoning. Learning how to use this knowledge purposefully and flexibly allows students to participate fully in an increasingly technology- and information-rich world of work.

Understand-Know-Do Overview

Understand

Understand describes the deep and enduring mathematical and statistical **big ideas** that students develop over phases 1–5.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Know describes the meaningful and important mathematical and statistical **concepts and procedures** through which students develop understanding of the big ideas.

Number | Mātauranga tau

Number focuses on the study of numerical concepts. People use numbers to represent quantities, estimate, and measure. We perform operations on numbers to calculate or compare. Throughout history, different number systems have been developed, reflecting practical and social needs.

Algebra | Taurangi

Algebra focuses on making and using generalisations to reason mathematically. It allows us to identify patterns and underlying mathematical relationships. These generalisations, patterns, and relationships can be represented and communicated using diagrams, graphs, and symbols (including variables). The algebra we use today was created and refined over thousands of years.

Measurement | Ine

Measurement focuses on the concepts and techniques that allow us to quantify phenomena, using appropriate units and systems of measurement. Countries around the world use both standard and non-standard units to measure tangible and intangible objects and quantities.

Geometry | Āhuahanga

Geometry focuses on visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects. Many cultures use tools and techniques derived from the natural world when exploring and describing objects and space.

Statistics | Tauanga

Statistics focuses on tools, concepts, and systematic processes for interpreting situations, using data and its context to understand uncertainty, make conjectures, and inform decision making. Statistical practices include considering the ethics of data collection and the responsibility of safely and securely handling data in different contexts.

Probability | Tūponotanga

Probability focuses on tools and concepts for quantifying chance, dealing with expectation, and using evidence to identify how likely events are to occur. People around the world have relied on and continue to rely on probabilistic thinking when making decisions.

Do

Do describes the **processes** that are fundamental to all mathematical and statistical activities and that underpin students' learning of the big ideas, concepts, and procedures.

Investigating situations | Te tūhura pūāhua

When we investigate situations using mathematics and statistics, we describe and explore them to build our understanding of them. When investigating, we need to decide which approaches, concepts, and tools to use and how to use them. We often begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts and procedures to solve problems and make sense of findings in context. We conclude by evaluating the investigation, which involves reflecting on the solutions and outcomes and our approaches and choices to determine whether they were reasonable, made sense in context, and could be improved on in future investigations.

Representing situations | Te whakaata pūāhua

When we represent situations mathematically and statistically, we use words or symbols and mental, oral, physical, digital, graphical, or diagrammatic ways to show concepts and findings. We can use representations to compare, explore, simplify, illustrate, prove, and justify, as well as to look for patterns, variations, and trends. Representing a situation in multiple ways enables a deeper and more flexible understanding and allows us to communicate with different audiences.

Connecting situations | Te tūhono pūāhua

When we connect situations using mathematics and statistics, we recognise and make links by noticing similarities and differences. Connecting helps us to understand the relationships between concepts and procedures in mathematics and statistics. This is important because number, algebra, measurement, geometry, statistics, and probability form a web of interconnected ideas and approaches that can be easier to remember and understand if the connections between them are clear. Connecting also involves linking mathematics and statistics to other learning areas and to a range of contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

When we generalise mathematical and statistical findings, we move from specific examples to general principles. We use the patterns, regularities, and structures that we find to make conjectures that might apply to other situations. Further investigation can test and refine these conjectures and determine if they apply in all cases. In statistics, we generalise by using trends and variation in data to make inferences and conjectures and to articulate and evaluate claims about similar situations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

When we explain and justify, we use mathematical and statistical ways of communicating and reasoning to share our ideas and to respond to the ideas, reasoning, and inferences of others. Explaining is how we communicate our inferences and conjectures, build arguments, and unpack stories from data. Justifying involves describing why decisions and findings are reasonable, taking into account limitations arising from assumptions and choices and the evidence on which findings are based.

Mathematics and statistics learning area structure

This section describes the structure of the mathematics and statistics learning area and how it changes over the five phases of learning.

Each phase has:

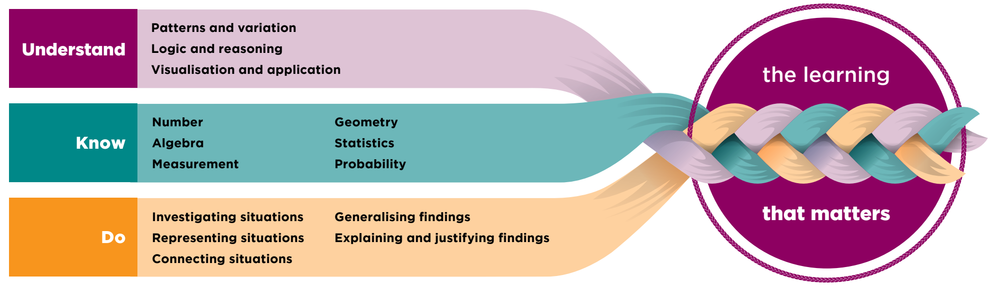
- a progress outcome describing what students understand, know, and can do by the end of the phase
- an introduction to the teaching sequence highlighting how to teach during this particular phase
- a year-by-year teaching sequence highlighting what to teach in the phase, along with teaching considerations for particular aspects of content.

Progress outcomes

The progress outcomes (one per phase) describe what students will understand, know, and be able to do by the end of the phase.

- **Understand** describes the big ideas that students develop from learning mathematics and statistics over phases 1 to 5. They help connect school mathematics and statistics with the wider world and represent the critical big-picture concepts of mathematics and statistics.
- **Know** describes the meaningful and important concepts and procedures in mathematics and statistics. They are broken down into six strands: number, algebra, measurement, geometry, statistics, and probability.
- **Do** describes the processes students use to represent and work with what they know and understand in mathematics and statistics. These processes are central to how students learn and apply mathematical and statistical knowledge. While there are small progressions in the processes from phase to phase, in general the increasing sophistication of their use comes from applying them to more advanced concepts and procedures.

It is through the interweaving of Understand, Know, and Do that students develop their conceptual understandings and procedural fluency, supporting success and bringing richness and meaning to mathematics and statistics for them.



Strands

Learning in each phase builds on learning from previous phases. In phase 1, the focus is on developing foundational concepts and procedures in all strands. In phases 2, 3, and 4, students expand their range of representations and their reasoning to work with increasingly complex concepts across all strands. In phase 5, year 11, students experience learning across the strands in a more holistic way, with measurement and geometry merged into a single strand. In years 12 and 13, the learning area allows for greater specialisation by dividing into the separate disciplines of mathematics and statistics.

	Phase 1 Years 0–3	Phase 2 Years 4–6	Phase 3 Years 7–8	Phase 4 Years 9–10	Phase 5 Year 11	Phase 5 Years 12–13
Number						
Algebra						**Mathematics**
Measurement						
Geometry						
Statistics						**Statistics**
Probability						

Teaching sequences

In years 1–11, the year-by-year teaching sequences are organised in line with the strands from Know. They describe the incremental teaching required each year as students work towards the progress outcome.

Some statements in the teaching sequences are repeated across multiple years, allowing more time for progression and consolidation. Not all statements are progressed each year; some topics start and others end, reflecting what is developmentally appropriate in learning in mathematics and statistics.

Each statement in a sequence varies in the amount of teaching time it requires. The learning area is designed to enable knowledge and procedures to be connected and taught together, so individual statements in a year sequence should be combined in ways that enhance learning.

The year-by-year content can be viewed both vertically and horizontally. The vertical view helps teachers know what to plan for across each year. The horizontal view allows teachers to follow the statements for one concept across several stages. This helps them understand the prior knowledge students may bring to their learning and helps them decide how to extend this learning. The horizontal view also helps teachers plan for mixed-level classes.

The teaching sequence statements are supported by 'teaching considerations'. These describe evidence-based practices and show how teachers can integrate the processes of Do to help their students develop conceptual and procedural knowledge.

Teaching guidance

Key characteristics of how people learn have informed the development of the mathematics and statistics learning area. These characteristics are:

- We learn best when we experience a sense of belonging in the learning environment and feel valued and supported.
- A new idea or concept is always interpreted through, and learned in association with, existing knowledge.
- Establishing knowledge in a well-organised way in long-term memory reduces students' cognitive load when building on that knowledge. It also enables them to apply and transfer the knowledge.
- Our social and emotional wellbeing directly impacts on our ability to learn new knowledge.
- Motivation is critical for wellbeing and engagement in learning.

All five characteristics are interconnected in a dynamic way. They are always only pieces of the whole, so it is critical to consider them all together. The dynamic and individual nature of learning explains why we see individual learners develop along different paths and at different rates.

The implications of these characteristics for teaching mathematics and statistics are described in this section, with more detail in the introduction to each phase and the 'teaching considerations' in the year-by-year teaching sequences.

The remainder of this section focuses on three key areas of teacher decision making:

- developing a comprehensive teaching and learning programme
- using assessment to inform teaching
- planning.

Developing a comprehensive teaching and learning programme

A comprehensive mathematics and statistics programme needs the following components:

- explicit teaching
- positive relationships with mathematics and statistics
- rich tasks
- communication in mathematics and statistics.

Explicit teaching

Explicit teaching is a structured, carefully sequenced approach to teaching. The sequencing of content is thought out and broken down into manageable steps, each of which is clearly and concisely explained and modelled by the teacher. Explicit teaching requires a high level of teacher-student interaction, guided student practice, and, when proficiency is achieved, independent practice.

Explicit teaching supports cumulative learning as new knowledge is built on what students already know. Teachers provide multiple opportunities for practising, reviewing, consolidating, and using previous learning alongside new learning.

Explicit teaching takes account of cognitive overload. With sufficient practice, new learning is transferred to long-term memory. This frees up working memory, opening up opportunities for extension, enrichment, and new learning.

Explicit teaching is strongly interactive – it is not simply teacher talk. It includes rich discussions between teachers and students and amongst students, to check on understanding. Teachers adapt the pace of their teaching in response to students' progress. They engage students in creative and challenging tasks to foster motivation and engagement.

Using materials and visual representations throughout explicit teaching supports students to develop conceptual understandings as they move towards more abstract forms of representation, such as equations. Teachers can reduce students' cognitive load by carefully considering the ways in which visual and written information are presented (e.g., how working and explanations are laid out) and by removing unnecessary information to focus on the key teaching and learning points.

Explicit teaching involves:

- connecting the current focus to previous learning
- providing concise, step-by-step explanations, accompanied by student input and discussion
- explaining, modelling, and demonstrating
- regularly checking for understanding and providing feedback
- providing opportunities for collaborative and independent practice.

Positive relationships with mathematics and statistics

Learning is enhanced when students succeed in and feel positive about their learning. If students feel anxious, they have fewer cognitive resources available for learning.

Positive relationships with mathematics and statistics are supported by teachers through:

- setting high expectations
- planning experiences that are accessible to every student and provide daily opportunities for success
- incorporating students' interests, cultures, and prior knowledge
- planning opportunities for students to explore and think critically
- supporting students to use mathematics and statistics to make sense of their world and address local, national, and global issues
- providing manageable challenges that encourage students to develop perseverance, reinforcing that conceptual understanding and procedural fluency develop with consistent effort
- increasing scaffolding and supports in response to anxiety as a result of cognitive overload
- valuing mistakes as an important part of the learning process.

Involving families in students' learning journeys and offering opportunities for collaboration support positive relationships with mathematics and statistics. Teachers also model such relationships by showing curiosity, persistence, and enjoyment, and by engaging in mathematics and statistics themselves.

Rich tasks

Rich tasks are meaningful problem-solving and investigation experiences, designed to invoke curiosity and engagement. They should relate both to mathematical contexts and wider contexts relevant to the communities, cultures, interests, and aspirations of students.

Rich tasks provide a motivational hook when exploring new concepts and procedures. They can also be used to consolidate concepts and procedures that have already been taught, to develop the mathematical and statistical processeses of Do, and to facilitate the transfer and application of learning to new situations. These experiences often allow students to decide how to approach the task, developing their agency, confidence, and motivation.

Teachers design rich tasks that are accessible to all students and offer different levels of challenge. They ensure that students are clear about the purpose of learning, and they consider the core requirements of the task as well as the range of possible responses. As students work on rich tasks, teachers plan opportunities for discussion, collaboration, and feedback. They are actively involved in monitoring, prompting, and questioning during the task, to encourage students to ask questions, test conjectures, make generalisations, and form connections.

Communication in mathematics and statistics

Students communicate throughout the learning process, both to develop conceptual understanding and to share their thinking and reasoning. Rich, extended interactions are pivotal to students' development of knowledge, processes, and dispositions in mathematics and statistics. Effective discussions build knowledge through sharing, active listening or attending, critiquing, questioning, and extending thinking and reasoning.

Rich interactions make students' reasoning visible. This helps teachers recognise how well students are developing mathematical and statistical processes and concepts, and it provides opportunities for teachers to identify misconceptions and correct them. These interactions also allow teachers to develop students' use of mathematical and statistical language, vocabulary, symbols, representations, and reasoning.

Using assessment to inform teaching

Assessment that informs decisions about adapting teaching practice is moment-by-moment and ongoing. Teachers use observation, conversations, and low-stakes testing to continuously monitor students' progress in relation to their year level in the teaching sequence. They ensure that they notice and recognise the development, consolidation, and use of learning-area knowledge by students within daily lessons, and that they provide timely feedback. They respond by adapting their practice accordingly. For example, they reduce or increase scaffolding and supports, paying particular attention to anxiety caused by cognitive overload. Formative assessment information can also be collected through self and peer assessment, with students reflecting on goals and identifying next steps.

In addition to daily monitoring, teachers use purposefully designed, formative assessment tasks at different points throughout a unit or topic to highlight the concepts and reasoning students use and understand. Teachers ensure such tasks are valid by addressing barriers to learning, so that every student is able to demonstrate what they know and can do.

When planning next steps for teaching and learning, teachers consider students' strengths and responses along with potential opportunities for further consolidation. Next steps could include:

- designing scaffolds to support students to access and enrich their learning
- providing opportunities for students to apply new learning
- planning lessons focused on revisiting, reteaching, or consolidating learning.

Providing timely feedback throughout the learning process and identifying and addressing misconceptions as they arise lead to the efficient and accurate development of learning-area concepts and promote further learning. Teachers can use feedback to prompt students to recall previous learning, make connections, and extend their understanding.

Planning

This section provides guidance on what to pay attention to when planning mathematics and statistics teaching and learning programmes. In every classroom, there are many ways in which students engage in learning and show what they know and can do. Using assessment information and designing inclusive experiences, teachers plan an 'entry point' to a new concept or procedure that every student can access. Students' interests and the school culture and community shape the planning, adding richness, creativity, and meaning to the programme.

Teaching and learning plans are developed for each year, topic or unit, week, and lesson and make optimal use of instructional time. The following considerations are critical when planning and designing learning:

- Develop plans using the teaching sequence statements for the year and knowledge of students' prior learning. Plan for all students to experience all the statements in the sequence for their year level.
- Map out a year-long programme composed of 'units' by looking for opportunities where statements from the teaching sequence can be taught together. These may be in the same strand or across several strands (e.g., statistics and measurement; algebra and geometry). Plan to weave together learning under Know and Do across the unit to build understanding of the big ideas.
- Order the units so that new learning builds on students' previous learning and connects over the course of the year. Consider the length of time allocated to specific strands and concepts across the year – some concepts may require more teaching time than others. Ensure the year's programme includes opportunities to retrieve, consolidate, and extend learning around previously taught concepts and processes. Regular opportunities to revisit learning within and across units and years supports students to develop procedural fluency with mathematics and statistics concepts. The shape of these opportunities will vary, depending on students' learning needs.
- Within unit or weekly plans, break down new concepts and procedures into a series of manageable learning experiences, so that students have several opportunities to develop understanding and fluency. In years 1–8, teach mathematics and statistics for an hour a day. Plan for a balance of explicit teaching (to introduce and reinforce learning) and rich tasks (to investigate a concept, support consolidation of previously taught concepts or procedures, and apply learning to new situations).
- Plan for inclusive teaching and learning at all times. Consider offering multiple methods of participating to all students so that they can engage in a variety of learning experiences and have multiple ways to show their progress. Design for equitable access in all learning opportunities. Identify and reduce barriers to learning, and plan universal supports that are available to all students.
- Use flexible groups within a lesson, based on the learning purpose for the lesson (e.g., working as a whole class for demonstration and discussion, in smaller groups to investigate a situation or solve a problem, and in pairs to explain thinking and findings). Provide opportunities for both individual and collaborative work, and enable students to determine when they need to work with others and when they need time and space to work independently.
- Teach students to use digital tools accurately, appropriately, and efficiently to support their purpose. Enhance teaching and learning with tools for calculating, representing graphs and shapes, and analysing data. While using digital technology is an important skill, students still need the ability to estimate, visualise, and reason, so that they can evaluate whether findings generated by a digital tool are reasonable and effective.

To support students who have not developed the prior knowledge needed for teaching sequence statements for their year or have not learnt everything they have been explicitly taught, teachers can use accelerative approaches. These are approaches that make year-level concepts and procedures accessible to students. They can include additional, targeted small-group teaching, the use of verbal and visual prompts, carefully chosen representations, and explicit teaching of problem-solving strategies.

Teachers can extend students who have developed deep conceptual understanding and procedural fluency for their year by using more challenging rich tasks and problem solving that allow the students to apply their understanding to unfamiliar situations. This also encourages the students to develop further generalisations and to strengthen their mathematical and statistical communication and reasoning.

Dedicated mathematics and statistics lessons

Depending on the purpose of the lesson, plan to include one or more experiences in each part (Getting started, Working, and Connecting and reflecting). As students are working, take time to notice, recognise, and respond to their learning.

Getting started	› Recall and connect to prior learning to provide a starting point for all students to access and understand new concepts or processes. › Introduce new concepts using a focus activity, group challenge, or task that activates prior knowledge and interests.
Working	› Provide whole-class, small-group, paired, or individual work opportunities for students to develop or apply concepts and procedures through investigations, tasks, or games. › Explicitly teach concepts and procedures by leading interactions that include explanations, demonstrations, questioning, short tasks, and discussion. Use clear and concise language, including correct mathematical and statistical vocabulary, and clear working layouts and notation. › Provide additional explicit teaching based on the learning needs of individual students. › Help students organise new knowledge in ways that connect with their prior learning – for example, by discussing connections, using graphic organisers, or carefully ordering concepts and procedures in relation to prior learning. › Support consolidation of knowledge with targeted practice and activities. For students early in the process of consolidation, these activities should be scaffolded and guided. As students develop understanding and fluency, they complete the activities with increasing independence. › Support students to retrieve and use previously taught concepts and procedures in connected ways, such as applying them while investigating situations.
Connecting and reflecting	› Clearly summarise and connect to the purpose of the lesson. › Review learning by discussing, sharing, and analysing the experiences of the lesson. › Make connections with prior learning, between mathematics and statistics concepts, with other learning areas, and with situations outside of the classroom. › Pre-teach to prepare students for the next lesson. › Highlight progress and examples of curiosity, resilience, and persevering through challenge.

The progress outcomes and teaching sequences for phases 1–3 are available in the Mathematics and Statistics Years 0–8 document.

Phase 4
Years 9–10

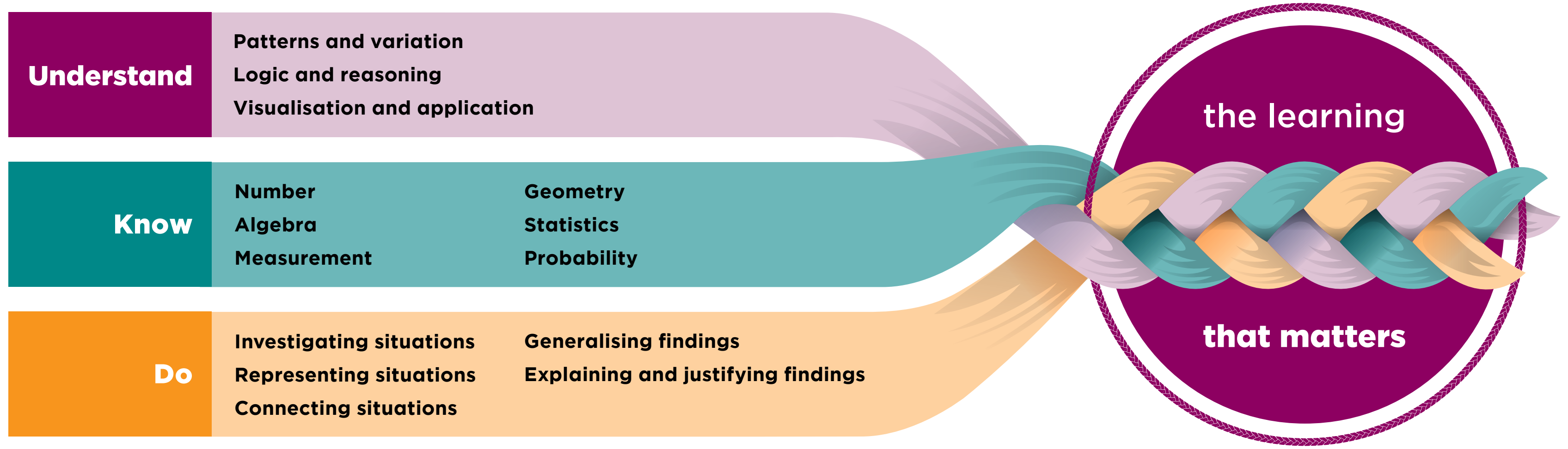
Progress outcome by the end of year 10

Having a purpose and being empathetic and resilient
Te whai ahunga, te manaaki i ētahi atu me te mau tonu ki te manawaroa

In phase 4, through their learning in mathematics and statistics, students deepen their understanding of the importance and usefulness of the learning area. They apply procedures in a variety of ways to purposefully investigate simple situations, recognising and valuing other perspectives. As they learn more complex concepts they develop resilience, which they draw on when seeking mathematical and statistical solutions.

Students use proportional reasoning to transform numerical quantities, measurements, and shapes, including right-angled triangles. By investigating patterns, they begin to generalise their understanding of tables, equations, and graphs and of how to connect the different representations. They extend their understanding of area, perimeter, and volume to work with a variety of 2D shapes, including circles, and 3D shapes, including prisms. They use data visualisations to investigate, represent, and explain patterns, trends, and variation, and they apply their knowledge to situations involving chance.

The phase 4 progress outcome describes the understanding, knowledge, and processes students have multiple opportunities to develop over the phase.



The phase 4 progress outcome is found on the following two pages.

Understand

As students build knowledge through their use of the mathematical and statistical processes, they deepen their understanding of the following.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Number | Mātauranga tau

By the end of this phase, students know that the order of operations is important when evaluating or forming expressions, and that grouped operations are always done first (e.g., situations under a square root, involving the numerator of a fraction, or inside brackets). Students know that there are an infinite number of rational numbers between any two numbers, and that these can be represented by terminating decimals, recurring decimals, or fractions. Non-repeating, infinite decimals are irrational numbers, and some of these are represented by special symbols, such as $\sqrt{2}$ and π. A number written in scientific notation has the form $a \times 10^k$, where $1 \le a < 10$ and k is an integer.

Students know that ratio concepts can be extended to proportionally compare two or more quantities. Ratios can show part-to-part or part-to-whole relationships and can be scaled up or down or simplified. A rate proportionally compares two quantities that have different units of measure; in rates, 'per' means 'for every' in everyday contexts.

Algebra | Taurangi

By the end of this phase, students know that the properties of operations (commutative, distributive, associative, inverse, and identity) and the order of operations apply to numbers and variables. When operating on or writing equations with fractions, fractions bigger than 1 are usually written as improper fractions. Any positive real number has two square roots: one positive and one negative. The principal square root is the positive root. The square root operation ($\sqrt{\ }$) refers specifically to the principal square root. There are 0, 1, or 2 real-number solutions to $x^2 = a$, where a is a number.

Measurement | Ine

By the end of this phase, students know that decimal measures are used for very small durations (milliseconds) but that the rest of time measurement uses a different system, based principally on 12 and 60. They know that the number of significant figures in a measurement is the number of digits that contribute to the degree of accuracy of the measurement. Resizing a shape changes its perimeter, area, or volume proportionally according to the dimensions of the units; linear metric conversions must be squared to convert area and cubed to convert volume.

Students know that in right-angled triangles there is a fixed relationship between the lengths of the three sides given by Pythagoras' theorem. The circumference and area of a circle can be approximated using knowledge of polygons and found exactly with formulae that use pi. Students continue to strengthen their use of metric units and conversions between different-sized units, and they develop the idea of derived units, which reflect the ratio between two different measurements. For example, when multiplying lengths or dividing volume by length, the result has a derived unit.

Geometry | Āhuahanga

By the end of this phase, students know that for all polygons, there is a generalisation for the sum of interior angles and the sum of exterior angles. In similar shapes, corresponding angles are equal and the lengths of corresponding sides are proportional. Congruent shapes are identical in shape and size. A set of points in a plane can be transformed by translation, reflection about a line, and rotation about a fixed point. When working with straight lines, angles between parallel lines and a transversal can be corresponding, co-interior, or alternate, and they have known relationships. A circle is the path traced out by a point moving in a plane and always a fixed distance (the radius) from the centre of the circle.

Statistics | Tauanga

By the end of this phase, students know that the collection, use, and storage of data needs to respect cultural sensitivities and obey laws about data and privacy. Different countries have different laws and protocols for data and privacy. It is not always possible to get data about everything or from everyone (the entire population), and there are ways of addressing this (e.g., by using sampling, the choosing of objects or individuals from a population to form a sample).

Students know that the statistical enquiry cycle (PPDAC) can be used to conduct data-based investigations that involve sampling from populations. When sampling from a population, the distribution for a variable varies from sample to sample. In experiments, the experimental units – where we collect the data from – can be people, animals, plants, objects, or non-physical entities. New variables can be created by combining and modifying existing variables. Uncertainty should be taken into account when making claims.

Probability | Tūponotanga

By the end of this phase, students know that a simulation is a way of modelling a chance-based situation, generally using digital tools. The statistical enquiry cycle (PPDAC) can be used to conduct chance-based investigations involving simulations. Simulations give experimental estimates of probabilities, and these, along with theoretical-model probabilities, are approximations of the true probabilities, which are never known. These probabilities are underpinned by assumptions that are not necessarily the same; for example, we assume that a coin is fair. Some variation between experimental and theoretical-model probabilities is normal.

Students know that there are three different types of models for chance-based situations: 'good model', 'no model', and 'poor model'. A 'good model' is fit for the purpose for which it is being used (e.g., the standard theoretical model for tossing a dice). With 'no model', there is no obvious theoretical model, and so we can only estimate probabilities and probability distributions via experiments (and these estimates can be used to build a theoretical model). A 'poor model' may be unreliable – for example, with spinning a coin, it may appear that heads and tails are equally likely; but a large number of trials shows that it is not a good model and a new model is required, using estimates from experiments.

Students also know that in compound events, events can be dependent or independent, and that elements of chance affect the certainty of results from observational studies or experiments.

Do

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it in a systematic and organised way. They can identify relevant prior knowledge, conditions, assumptions, constraints, and relationships. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate relationships, objects, and ideas.

Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They make connections to ideas in other learning areas and with diverse cultural, linguistic, and historical contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others. They present reasoned, coherent explanations and arguments for an idea, solution, or process.

Phase 4 Years 9–10

Teaching sequence

Having a purpose and being empathetic and resilient
Te whai ahunga, te manaaki i ētahi atu me te mau tonu ki te manawaroa

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the fourth phase of learning at school.

Throughout phase 4, encourage analytical thinking, perseverance, and an appreciation for the complexity of mathematical concepts. Challenge students to engage with mathematical problems and to critically reflect on both their own and others' reasoning. Encourage them to question assumptions, seek alternative approaches, and explore multiple solutions. Foster an environment of collaborative inquiry where students are expected to take an active role in discussions, supporting and encouraging each other while reflecting on one another's strategies and outcomes. Empower them to make informed decisions about their mathematical explorations, from choosing appropriate methods to justifying their conclusions. This will promote their sense of ownership of their learning journey.

Explicit teaching

- Use worked examples and break down new learning into clearly explained, manageable steps. Show students efficient written and mental methods. Encourage them to identify their own errors, misconceptions, and missing steps and to use recently learned techniques to correct these, with teacher guidance. Support them to develop critical analysis and reasoning skills.
- Plan for students to actively recall learning, practise new procedures, and make connections with prior learning. Provide regular opportunities to practise, so that students maintain their automatic recall of facts and continue to develop procedural fluency and reasoning.
- Support students to consolidate what they have learned by repeating a process or task you have demonstrated, so that they develop confidence in applying skills in unfamiliar situations.

Positive relationships with mathematics and statistics

- Select high-interest tasks that provide a sufficient level of challenge and connect to students' experiences in the classroom and the wider world.
- Encourage perseverance by demonstrating the value of trying alternative approaches.

Rich tasks

- Design investigations where students experience rich mathematical situations, as well as investigations where they use their findings to make decisions in their lives (e.g., planning a holiday). Support students to identify appropriate questions, as well as the mathematical and statistical concepts, procedures, and representations they will need.
- Design tasks that have multiple entry and exit points and more than one solution or pathway. Use contexts that are familiar to students and that connect with other strands or learning areas. Carefully choose questions and statements that invite students to focus on the concepts underpinning the learning purpose of the task.
- Support students to convert their wonderings or observations into conjectures and to provide working or counterexamples to prove or disprove them. Encourage them to check if their conjectures always work, if there are times when they don't work, and if they can be extended.
- Encourage students to identify the key components from a situation or problem. Ask students to represent them with the correct notation and to then find an efficient strategy to solve the problem, representing their working and reasoning using diagrams, materials, and digital tools as appropriate.
- Check in at specific points throughout an investigation and support students to progress their work. Respond as appropriate with specific, targeted teaching of the mathematical or statistical skills needed to progress the investigation.
- Encourage students to try a range of procedures and processes in their investigations. Guide them towards selecting the most appropriate process for the problem. Help them to recognise that a first approach not working is common in mathematics and statistics, and that they should then try another approach.

Communication in mathematics and statistics

- Set up opportunities for students to actively listen, reflect, and build on each other's thinking and learning. Initiate, shape, and sustain purposeful discussions, and model productive challenges. Over the phase, encourage students to use evidence to justify their claims and findings. Ensure that teacher talk is balanced with opportunities for students to actively particate in and lead discussions.
- Encourage students to select and use representations that best support the learning purpose, including graphs, tables, and equations. Help them to develop confidence in using higher-order mathematical notation and vocabulary.
- Encourage students to visualise in order to mentally represent and manipulate relationships, shapes, and quantities.
- Encourage students to be respectful of each other's views. They should be prepared to argue their point of view or thinking and to explain their reasoning and justifications, while respecting others doing the same.

Number

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Number structure	› record, compare, and order whole and decimal numbers using scientific notation (e.g., 3.14×10^3)	› record, compare, and order whole and decimal numbers, including those represented by negative powers in scientific notation (e.g., "Which is larger, 7.35×10^{-3} or 0.008?")	Demonstrate scientific notation for numbers where the coefficient is between 1 and 10 and the powers of ten are integer exponents only. **Represent** very large numbers and very small numbers with scientific notation. Explore the use of scientific notation in the mathematics strands (e.g., limits of accuracy in measurement, or significant figures in number) and in other learning areas (e.g., science). **Investigate** contextual situations that students are familiar with (e.g., other learning areas' use of very small or large numbers, or population growth in the news).
	› identify special properties of numbers, including cube roots of cube numbers up to 1,000		Demonstrate exponential growth of whole numbers, **explaining** the relationship between base, power, and root (e.g., $2^3 = 8$, $\sqrt[3]{8} = 2$). **Represent** cube numbers and other exponential numbers in exponential form, **generalising** that the base is the root (e.g., $2^5 = 32$, $\sqrt[5]{32} = 2$).
Operations	› use rounding and estimation to predict results and to check the reasonableness of calculations	› use rounding and estimation to predict results and to check the reasonableness of calculations	Use word problems and practical situations outside the classroom, ensuring that the magnitude of numbers involved is appropriate to the year level.
	› round to the degree of precision required for the context	› round to the degree of precision required for the context	**Connect** with measurement and other practical situations.
	› express remainders as fractions or decimals, depending on the context		**Represent** and make sense of remainders as fractions, as decimals, and when rounded to the nearest whole number, using number lines and diagrams.

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Operations	› evaluate expressions with positive whole number exponents, using the order of operations	› evaluate expressions using the order of operations (e.g., $\frac{2 - 3^2}{2}$)	Demonstrate a mnemonic such as GEMA (grouped, exponents, multiplicative, additive), and common errors. **Investigate:** › the order of operations – why it is used, and what happens when it is not followed › situations in familiar contexts, to reinforce students' skills in practical situations (e.g., using number operation skills in measurement and algebra problems and in financial mathematics) › operations that involve square roots with exponents (e.g., $\sqrt{3^2 + 4^2} \neq 3 + 4$) › operating on numbers with whole-number exponents, including powers with negative numbers (e.g., $(-1)^4 = 1$).
	› add and subtract integers, multiply integers using repeated addition or subtraction, and divide integers by reversing a multiplication (e.g., $-12 \div 3 = -4$, since $3 \times -4 = -12$)	› add, subtract, multiply, and divide positive and negative numbers, including fractions and decimals	**Represent** operations on number lines and with two-sided counters, connecting with practical situations. **Investigate:** › multiplicative inverses with negative numbers (e.g., $\frac{1}{-b} = -\frac{1}{b}$) › how multiplication and division as inverse operations extend to negative numbers (e.g., $3 \times -2 = -6$) and families of facts › practical situations outside the classroom that use integers (e.g., sea level or temperature) › why division by a fraction is equivalent to multiplication by its reciprocal. **Generalise** relationships between positive and negative integers using the commutative, associative, and distributive properties. **Explain** and **justify** the effect of dividing by numbers between 0 and 1.

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Rational numbers	› identify, read, write, represent, compare, order, and convert between fractions, decimals, and percentages	› identify, read, write, represent, compare, order, and convert between fractions, decimals, and percentages	**Represent** fractions, decimals, and percentages on number lines and 100s squares. Demonstrate the use of flow diagrams in conversions. **Investigate:** › fractions, decimals, and percentages as equivalent representations of the same values › practical situations where fractions, decimals, percentages, rates, and ratios are used.
	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	Use factors trees to **investigate** highest common factors and lowest common multiples. **Explain** that 'improper' is just a technical term for 'top-heavy' fractions, which are a useful format for working with fractions.
	› find a fraction or percentage of a number, and increase or decrease a number by a fraction or percentage	› calculate the percentage increase or decrease between two numbers (e.g., "What is the percentage increase between 50 and 75?")	Demonstrate how to calculate a percentage increase or decrease with a single decimal multiplier. **Investigate** common and expected uses for percentage increases and decreases outside the classroom (e.g., discounts, mark-ups, and interest).
	› find the whole amount, given a fraction or percentage (e.g., "20% of an amount is 30, what is the original amount?")	› find the whole amount, given a fraction or percentage (e.g., "15% of an amount is 27, what is the original amount?")	**Represent** backwards and forwards situations using flowcharts and bar models. **Connect** finding an original amount with forming and solving a linear equation (e.g., $\frac{15}{100}x = 27$).
	› divide fractions by whole numbers, and add, subtract, and multiply fractions		**Investigate** reciprocals, equivalent fractions, and operations using number lines.
	› divide decimals by whole numbers, and add, subtract, and multiply decimals		**Represent** decimal numbers using place-value materials. **Connect** operations with decimals to operations with whole numbers and operations with fractions. **Explain** and **justify** the effect of multiplying by numbers between 0 and 1.

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Rational numbers	› use ratios to share in unequal proportions, and find equivalent ratios and rates by scaling up or down	› compare and use ratios and rates	**Connect** ratios to fractions, explaining how the parts of the ratio can become the numerators of corresponding fractions, and the sum of the parts the denominator. **Investigate** practical situations that use fractions and ratios.
Financial mathematics	› calculate profit, loss, and discounts as absolute quantities relative to an original amount	› convert NZ dollars into other currencies (and vice versa) and give examples of when this is useful	**Connect** to operations that use decimals and, in year 10, operations that use rates. **Investigate** the number of decimal places and rounding in cash versus electronic transactions.
	› calculate simple interest and GST on dollar amounts (e.g., "Find 15% GST on $432").	› find proportions of costs (e.g., the price of 400 g of an item when given the cost per kilogram).	**Connect** to operations that use fractions, decimals, and percentages. **Investigate** problems that involve accruing interest or budgeting to make a savings target.

Algebra

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Equations and relationships	› simplify, expand, and factorise algebraic expressions involving sums, products, differences, and positive integer powers	› simplify, expand, and factorise algebraic expressions	**Represent** operations with algebraic expressions visually, using materials such as algebra tiles, array models, and expansion tables. **Investigate** visual patterns that lead to different ways of representing algebraic expressions, including factorised, expanded, and simplified forms, and show their equivalence.
	› form and solve linear equations with rational number solutions and linear inequalities with positive coefficients	› form and solve linear equations and inequalities with rational number solutions, giving exact and rounded solutions	Demonstrate algorithms for solving equations. Consolidate learning with puzzle squares. **Explain** how the solution to an equation has been reached.

	During year 9 *Informed by prior learning, teach students to:*	**During year 10** *Informed by prior learning, teach students to:*	**Teaching considerations**
Equations and relationships	› find the value of an expression or formula given the values of its variables, and rearrange simple formulae	› substitute into, rearrange, and simplify expressions or formulae	Demonstrate the use of variable values in practical situations that involve familiar formulae (e.g., areas or temperature conversions). **Connect** to other learning areas that use formulae (e.g., science, technology, or social sciences).
	› create, interpret, graph, and express linear relationships in the form y = mx + c, where m is the constant rate of change (gradient or slope) and c is the y-intercept	› create, interpret, graph, and express linear relationships in more than one form	Demonstrate different forms of linear equations using points, gradient-and-intercept, and intercept-and-intercept methods of forming equations and graphing linear relationships. **Investigate** relationships between equations and graphs, using manual and digital tools.
	› recognise the relationships between elements in a non-linear pattern, write an equation using variables and algebraic notation to represent the rule for the pattern (e.g., $y = x^2 + c$), and use the equation to make conjectures	› determine the effect on XY graphs of changing the coefficient of x^2 and the fixed value, for a range of quadratic relationships of the form $y = ax^2$ or $y = x^2 + c$, where a is a positive whole number and c is an integer	**Investigate** changes to parabolas that result from changes to their equations, using manual and digital tools. **Generalise** patterns using words, formulae, and graphical representations.
Algorithmic thinking	› test and improve algorithms that use sequence (determining the order of steps), selection (choosing steps), and iteration (repeating steps).	› design, create, test, and refine algorithms that use sequence, selection – using logical operators, such as equals or not equals – and iteration.	Introduce students to basic coding language and associated digital tools. Use the formula function of a spreadsheet to **investigate:** › the effect of changing one or more variables on results (i.e., cell values) › the modelling of practical situations that involve measurements and values that change. **Connect:** › iterative thinking to other strands (e.g., the relationship between the perimeter and area of a rectangle in measurement) › with other learning areas to provide opportunities for students to engage with algorithmic thinking and coding. **Investigate** a range of digital tools to explore different methods of approaching algorithmic thinking digitally.

Measurement

	During year 9 *Informed by prior learning, teach students to:*	**During year 10** *Informed by prior learning, teach students to:*	**Teaching considerations**
Measuring	› estimate, calculate, convert, and accurately represent measurements using significant figures	› estimate, calculate, convert, and accurately represent measurements using significant figures	Have students practise taking accurate measurements from diagrams or physical objects using appropriate tools (e.g., a ruler, protractor, scales, or calculator). **Represent** all written measurements with their correct units. Provide situations in which students are engaged in the process of collecting measurements in order to find lengths and angles or solve a practical problem.
	› select and use appropriate measurement units for a given context, if necessary converting between metric units using appropriate prefixes	› convert between metric units, and use appropriate prefixes in the metric system (e.g., tera-, giga-, mega-, kilo-, centi-, milli-, micro-, and nano-)	Use flow diagrams to demonstrate conversions, showing that the larger the new unit, the smaller the resulting number. Explore how measurements are made in different disciplines, including preferences for units, (e.g., the building industry uses mm and m, whereas fashion uses cm). **Connect:** › with how other learning areas use measurement › between different units, including rates.
	› use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle from a given diagram	› use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle	Demonstrate the use of formulae in practical measuring. **Investigate** the properties of triangles using Pythagoras' theorem. **Connect** the properties of right-angled triangles with geometric reasoning about triangles and practical problems outside the classroom.
	› find speed, distance, and time, given any two of the measurements and recognising that distance over time is a derived measure	› find speed, distance, and time, given any two of the measurements and recognising that distance over time is a derived measure	**Investigate** the relationships between speed, distance, and time in practical situations. **Connect** finding the value of variables in the speed formula with solving algebraic equations and multiplication and division operations.

	During year 9 *Informed by prior learning, teach students to:*	**During year 10** *Informed by prior learning, teach students to:*	**Teaching considerations**
Measuring	› reason about duration using different units of time and fractions of units of time, including decimal fractions of milliseconds where appropriate	› reason about duration using different units of time and fractions of units of time, including decimal fractions of milliseconds where appropriate	Demonstrate duration using stopwatches and different ways of measuring and comparing it (e.g., elapsed time in timetables). **Connect** with: › other metric units of measurement, exploring the differences between them and time measurement › other systems of time measurement (e.g., the lunar calendar). **Investigate** ways of sorting and ordering time, including time conversions, the different divisions of time, and the history of why we measure time using a non-metric system. Engage students in activities about time differences by exploring different time zones and calendars.
Perimeter, area, and volume	› find the area of parallelograms, trapeziums, and kites, relating the formulae used to the formula for a rectangle › find the circumference and area of circles	› find the surface area and volume or capacity of prisms and cylinders	Explore relationships and similarities between formulae for area and perimeter. **Generalise** from specific shapes to more general shapes, (e.g., applying findings for a rectangular prism to a prism with any face shape). Demonstrate the use of formulae in practical situations. Engage students in practical exercises that allow them to explore and make generalisations about shapes, objects, and measurement units. **Investigate** the connection between the number of dimensions of a shape and the exponent of the corresponding measurement unit.
	› scale a shape by a factor, and find the perimeter, area, or volume of the scaled shape.	› scale a shape by a factor, and determine the scale factor for the scaled shape's area or volume.	**Investigate** practical situations to demonstrate how the scale factor of a shape's side relates to the scale factor for the scaled shape's area or volume.

Geometry

	During year 9 *Informed by prior learning, teach students to:*	**During year 10** *Informed by prior learning, teach students to:*	**Teaching considerations**
Shapes	› identify and describe parts of a circle (e.g., a chord; the diameter, radius, and circumference) and how they relate to each other	› use the properties of similarity in 2D shapes, including right-angled triangles, to find unknown lengths and angles	**Investigate** the relationships between the radius, diameter, and circumference of a circle. **Explain** when and how these involve the constant π. Demonstrate the parts of a circle (at year 9) and the properties of similarity (at year 10) using shapes, objects, diagrams, and measurement tools (e.g., rulers or string).
	› reason about unknown angles in situations involving intersecting lines, parallel lines, and transversals	› reason about unknown angles in situations involving parallel lines and transversals and the interior and exterior angles of polygons	Explore unknown angles using digital tools or diagrams with measuring tools (e.g., rulers or protractors). **Connect** with algebraic thinking when solving problems that involve unknown angles, practical measurement, and composite shapes. **Generalise** from specific physical or digital examples of circles, polygons, and sets of lines to explore rules about unknown angles, using algebra. Use geometry rules to **explain** and **justify** solutions to problems, exploring where the rules have come from and why they work.
Spatial reasoning	› represent and construct 3D shapes, including rectangular and triangular prisms, from nets, plan views, and isometric drawings		Use measurement tools to create physical and digital models. **Connect** with practical measurement and technical drawing, introducing design and visual elements.
	› transform 2D shapes in the XY plane by translation, reflection across a given mirror line, and rotation about a given point by a multiple of 90 degrees	› transform 2D shapes, including composite shapes, by resizing them by any scale factor.	Predict and test transformations using physical and digital drawing tools (grid paper, mirrors, tracing paper, and computer-aided design programmes).
Pathways	› use map scales and compass directions (e.g., N30°W) to interpret and communicate distance and direction from one location to another.		Use map reading, orienteering, and geocaching. **Connect** with the practical use of maps and navigation and tools within digital maps. **Investigate** the world outside the classroom through practical activities with maps and scales.

Statistics

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Problem	› investigate, using multivariate datasets from observational studies, summary, comparison, time-series, and relationship situations by: – posing an investigative question about groups of interest – making conjectures or assertions about expected findings	› investigate, using multivariate datasets from observational studies, summary, comparison, time-series, and relationship situations by: – posing an investigative question, in particular a summary or comparison question about a population, or a time-series or relationship question about a group of interest – making conjectures or assertions about expected findings	**Investigate** an area of interest, starting broadly and fine-tuning to arrive at a specific investigative question. Demonstrate how to pose an **investigative** question for different investigative situations: › summary and comparison situations, including the variable and groups of interest (at year 9) or population of interest (at year 10) › relationship and time-series situations, including the variable and group of interest. **Connect** the investigative question with conjectures about expected findings.
Plan	› plan how to collect or source data to answer the investigative question, including: – identifying the variables needed to answer the question – planning how to make valid and reliable measures for the variables (when collecting data) or finding out how they were measured (when sourcing data) – identifying the group of interest or who the data was collected from *(continued on the next page)*	› plan how to collect or source data to answer the investigative question, including: – identifying the variables needed to answer the question – planning how to make valid and reliable measures for the variables (when collecting data) or interrogating sourced datasets to understand what the variables measure and how they were measured – identifying the group or population of interest or (for sourced data) 'who' the data was collected from *(continued on the next page)*	**Explain** and **justify** methods of data collection, 'who' to measure, and what and how to measure, in order to answer the investigative question. Support students to pose data-collection and survey questions and **justify** how they will collect the data required to answer the investigative question. **Investigate** secondary datasets to **explain** their variables and group(s) or population(s) (at year 10) of interest. Reinforce how to apply ethical practices to data collection and use, including **explaining** what informed consent means and how to use data privacy protocols. *(continued on the next page)*

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Plan *(continued from previous page)*	– using interrogative questions to check that ethical practices are considered throughout the entire enquiry cycle (including testing data collection and survey questions with peers)	– determining if the investigative question is about a group or a population – if a population, determining what sample size is needed to make a decision about what is happening in the population – using interrogative questions to check that ethical practices are considered throughout the entire enquiry cycle	**Investigate:** › sample sizes needed to make a call about what is happening in the population (year 10) (about 1000 for categorical variables and about 30 for numerical variables) › how to determine and define the variables needed and plan to collect valid and reliable measures for them › possible misinterpretations of survey and data-collection questions and how this can lead to unreliable data.
Data	› collect or source data, including: – checking its validity and making simple edits if appropriate – creating a data dictionary (for collected data) or identifying the metadata (for sourced data) – recategorising or constructing new variables if needed to support answering the investigative question	› collect or source data, including: – checking its validity and making simple edits if appropriate – using random samples to collect or source data about a population (for summary and comparison situations) – creating a data dictionary (for collected data) or identifying the metadata (for sourced data) – recategorising or constructing new variables if needed to support answering the investigative question	Support students to use a variety of methods for collecting or sourcing the data, including: › making measurements › observing outcomes › using survey forms › using secondary data sources › using random samples (at year 10). Demonstrate how to record the data using a variety of tools – for example: › spreadsheets › recording sheets › data dictionaries for primary data › metadata for secondary data. Support students to identify errors in the data, **connecting** to the context, **explaining** why they are errors, and, if appropriate, cleaning the data using appropriate techniques. Demonstrate how to recategorise variables and construct new variables.

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Analysis	› create multiple data visualisations for the investigation, providing global and local views › describe and reason from the data visualisations, including: – identifying relevant features in distributions – interweaving the context in descriptions of distributions – drawing an 'eyeballed' line or curve of best fit to predict possible y values (for the response variable) for given x values (for the explanatory variable) in relationship situations, where appropriate	› create multiple data visualisations for the investigation, providing global and local views › describe and reason from the data visualisations, including: – identifying relevant features in distributions – interweaving the context in descriptions of distributions – making an informal inference about what might be happening in the population, based on visual considerations in comparison situations – drawing an 'eyeballed' line or curve of best fit to predict possible y values (for the response variable) for given x values (for the explanatory variable) in relationship situations, where appropriate – making informal conjectures about group membership (e.g., in relation to socio-economic status or educational attainment)	**Represent** data using dot plots, bar graphs, frequency tables, box plots, histograms, paired plots, time-series graphs, two-way tables or graphs, and scatter plots. Transition from creating these by hand to using digital tools. During analysis, support students to identify features, using: › fractions, proportions, and percentages › the mode, median, mean, and distributional shape › maximum and minimum values › the interquartile range, lower quartile, upper quartile, middle 50%, and range › descriptions such as more than, less than, at least, at most, between A and B › outliers, clusters, gaps, joint and conditional proportions › an eyeballed line or curve of best fit, seasonal patterns, and the long-term trend › decision trees. Check data visualisations for misleading features or information. Describe what is and is not seen in data visualisations, recognising that data are numbers with context and that the context includes variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables. When comparing data visualisations of the same variable for different groups, look at similarities and differences. In year 10, make an informal inference about what might be happening in the population, using the 'Making the call Year 10' decision guide.

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Conclusion	› communicate findings in context to answer the investigative question, using evidence from analysis › provide possible explanations for findings › reflect on conjectures or assertions › evaluate the approaches taken in the different phases of the enquiry cycle	› communicate findings in context to answer the investigative question, using evidence from analysis and with an awareness of variability › provide possible explanations for findings › reflect on conjectures or assertions › evaluate the approaches taken in the different phases of the enquiry cycle	Have students practise answering the investigative question using evidence from their analysis and with an awareness of variability. Explore relevant explanations or interpretations of findings connected to the context of the situation, including the group or population of interest and the variable or variables of interest. Support students to prepare and present succinct findings and to evaluate and recommend any changes to the processes they used. Ask them to explain whether or not their findings align with their initial conjectures or assertions, if what was found makes sense with what is known about the situation, and if there are any limitations to the findings. Ask students to consider, as a result of the statistical investigation, what else they could **investigate**, starting the PPDAC cycle again.
Statistical literacy	› evaluate the data-collection methods and findings of others' statistical investigations to see if their claims are reasonable › critically consider data visualisations to see if they support or misrepresent the data.	› evaluate others' statistical investigations to see if their claims are reasonable › critique others' findings and claims by closely examining all phases of the statistical enquiry cycle › critically consider data visualisations to see if they support or misrepresent the data.	Explore existing statistical reports, graphs, visualisations, and claims in order to determine their validity. Use readily available media (e.g., news media or social media) as sources. **Investigate** misleading data visualisations, match data visualisations with statements made, and check the claims made by others. Use interrogative questions to evaluate data-collection methods that involve, for example, ethical considerations, analysis processes, visualisations, and statements of findings.

Probability

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Probability investigations	› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by: – posing an investigative question – systematically listing outcomes for the sample space – constructing a model using theoretical or conjectured probabilities – deciding on the number of trials, the tools to be used, and the recording method – running simulations and recording data – creating data visualisations for the distribution of observed outcomes from the experiment and the distribution of possible outcomes for theoretical probability models – describing what these visualisations show – finding probability estimates for the different outcomes – proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists (e.g., tossing a non-regular 3D shape) – identifying similarities and differences between their findings and those of others – reflecting on anticipated outcomes – identifying and giving potential reasons for similarities and differences between findings from the probability experiment and associated theoretical probabilities	› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by: – posing an investigative question – systematically listing outcomes for the sample space – constructing a model using theoretical probabilities, probabilities based on past data, or conjectured probabilities – deciding on the number of trials, the tools to be used, and the recording method – running simulations and recording data – creating data visualisations for the distribution of observed outcomes from the experiment and the distribution of possible outcomes for theoretical probability models – describing what these visualisations show – finding probability estimates for the different outcomes – proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists (e.g., tossing a drawing pin) – identifying similarities and differences between their findings and those of others – reflecting on anticipated outcomes – identifying and giving potential reasons for similarities and differences between findings from the probability experiment and associated theoretical probabilities	**Investigate**, using the statistical enquiry cycle, everyday chance-based situations, patterns in possible outcomes, theoretical and experimental distributions, and relative frequencies from data investigations. **Represent:** › probability outcomes (theoretical and experimental) using lists, tables, tree diagrams, tally charts, visualisations of distributions, words, numbers, and digital tools › probability models. **Explain** how to describe and use probability concepts – for example: › outcomes, events, and the sample space › trials, simulations, models, and theoretical and experimental probability › with and without replacement, independence and dependence, joint and conditional probabilities › the law of large numbers › probability estimates, theoretical probability, and probability distributions › chance, randomness, and variation. **Connect:** › anticipated distributions with theoretical distributions and experimental distributions, linking representations (e.g., two-way tables) with distributions, words, and numbers › observed data, models, and simulated data.

	During year 9 *Informed by prior learning, teach students to:*	During year 10 *Informed by prior learning, teach students to:*	Teaching considerations
Critical thinking in probability	› identify why claims about chance-based situations might not be valid, and consider improvements	› identify why claims about chance-based situations might not be valid, and consider improvements	Explore existing claims about known chance-based situations and discuss where they came from, how valid they are, and what might be done to improve them.
	› evaluate the reasonableness of others' chance-based claims.	› engage in critical thinking for chance-based situations by: – recognising potential issues in the assumptions of a theoretical probability model that has been applied – evaluating the reasonableness of others' claims – explaining whether a claim is misleading and why and how probability may have been used in a particular way to arrive at the claim (e.g., in advertising).	Explore known chance-based situations, using logical reasoning to identify assumptions, problems, and the validity of claims. Provide opportunities and support for students to match the results of chance-based investigations with statements made about them, and to check others' claims about chance-based investigations. Evaluate statements made by others about their findings from chance-based investigations, using interrogative questions about how their probability experiment was conducted and their analysis of results.

The language of mathematics and statistics: Phase 4

	Year 9 *Students will know the following new words:*	**Year 10** *Students will know the following new words:*
Number	› cube root › GST › loss, profit › original amount › scientific notation › simple interest	› currency › percentage change
Algebra	› exponential › factorise › form › gradient, slope › intercept › iteration › linear equation or relationship › rate of change › rational number › selection	› operator › quadratic equation or relationship › rearrange
Measurement	› circumference, diameter, radius › derived measure › scale factor › significant figure	› surface area
Geometry	› alternate, co-interior, or corresponding angles › centre of rotation › chord › intersect › isometric › transversal	› centre of resizing or enlargement › similarity and congruence
Statistics	› decision tree › explanatory variable › line or curve of best fit › metadata › observational study › recategorising › response variable › variability	
Probability	› simulation	› assumption › limitation

Phase 5 Years 11–13

Progress outcome by the end of year 13

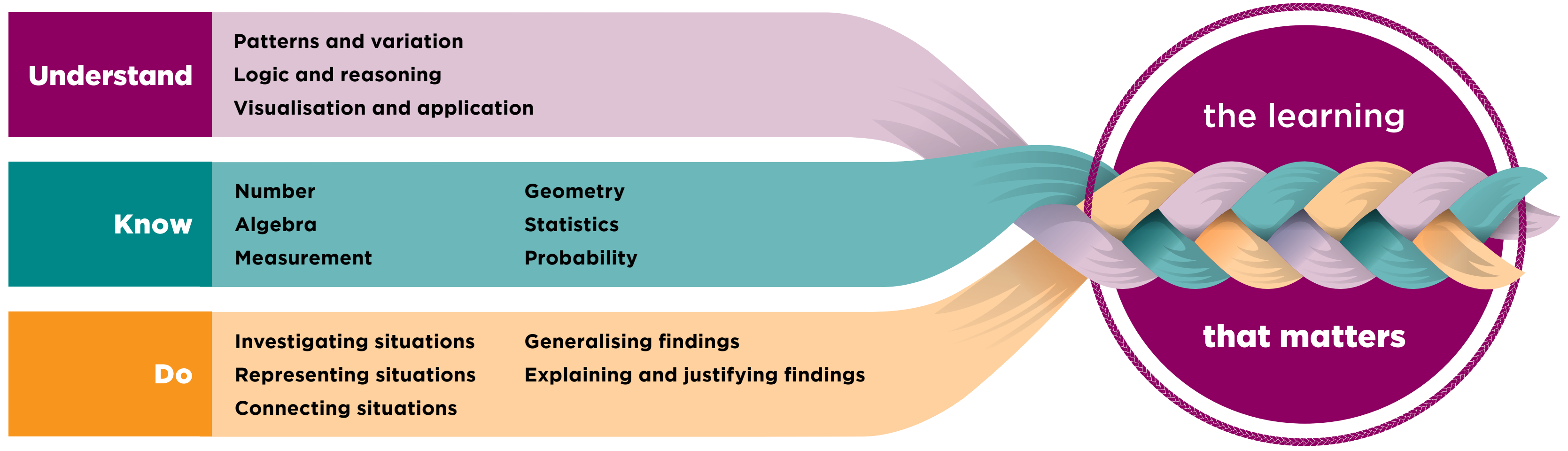
Navigating pathways and developing agency to help shape the future
Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō

In phase 5, learning becomes more specialised over the phase as students plan their pathways beyond school.

Students explore how the mathematics and statistics they are learning are used as tools in local and global contexts. They apply increasingly specialised techniques across the stands of mathematics and statistics to visualise and investigate complex situations, including exponential growth and decay, measuring physical objects or motion, elements of chance, and multivariate data. They investigate patterns and variation in mathematical functions and statistical data. They use logic and reasoning to provide comprehensive explanations, develop mathematical proofs, and evaluate statistical claims.

Students have opportunities to further develop and use what they have learnt in years 1–10 in a range of NCEA subjects. At years 12 and 13, the learning area mathematics and statistics separates into two separate disciplines (see page 9).

The phase 5 progress outcome describes the understanding, knowledge, and processes students have multiple opportunities to develop over the phase.



The phase 5 progress outcome is found on the following two pages.

Understand

As students build knowledge through their use of the mathematical and statistical processes, they develop a deep understanding of the following.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical-thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Mathematics | Pāngarau

By the end of this phase, students know that mathematical modelling involves investigating the relationships and behaviours of quantities in physical, economic, social, and everyday contexts. It is used to analyse applied situations and to make informed decisions, starting with forming assumptions. It also involves creating and solving equations and interpreting, evaluating, and refining results using the context. Digital technology plays a key role in mathematical modelling by enhancing visualisation, efficiency, and accuracy.

Students know that when calculating, they should use the appropriate form of numbers for the situation. Before and after calculating, they should use estimation to check the reasonableness of results. They give solutions with the appropriate degree of accuracy for the situation. When solving an equation, they will often rearrange it so that it equals zero. This allows for the use of the zero product property, which states that if two expressions multiply to be zero, then one or the other or both are zero.

Students know that the complex-number system is an extension of the real-number system and incorporates the square root of negative one. A complex number can be written in rectangular form, polar form, and exponential form.

By the end of this phase, students know that functions are relationships or rules in which each member of an input set maps to a single output. Functions can be represented by ordered pairs, tables, XY graphs, and algebraic equations, and they can be described in words for contextual situations. A function can undergo a variety of transformations with predictable effects on its graph. When working with functions, care needs to be taken that the distributive property is limited to expanding multiplication over addition and is not applied to expressions with powers and roots. Optimising a function enables the determination of its maximum or minimum value using various mathematical techniques (e.g., XY graphs, tables of values, linear programming, and calculus).

Students know the following key facts about specific functions:

- There can be many equivalent equations for expressing a linear function; different forms are appropriate for different situations, including in other learning areas.
- In a polynomial function, $x = a$ is a root if and only if $(x - a)$ is a factor of the polynomial.
- Exponential and logarithmic functions are inversely related.
- The standard unit of angular measurement is a radian, which is geometrically related to the length of the radius of a circle. A point on a unit circle at an angle of θ in standard position is represented by the coordinates $(\cos \theta, \sin \theta)$. This demonstrates the periodic and symmetric nature of the sine and cosine functions, visually and algebraically.

Students know that calculus is the study of how things change. The derivative of a function at a point is the gradient of the tangent line to the curve at that point. The derivative of a function can be interpreted as its rate of change. Tangent lines are local approximations of a function. Near a specific point, the tangent line and the function have approximately the same gradient and graph. In the graph of a function, a local extremum or a change in concavity can occur at points where the gradient of the tangent line is 0 or undefined. The indefinite integral of a function is a general antiderivative that includes a constant of integration. The derivative of the indefinite integral is the original function. The definite integral gives the net signed area bounded by a curve and the x-axis over an interval. Kinematics enables us to describe the motion and direction of movement of objects within closed systems in terms of displacement, velocity, and acceleration.

Statistics | Tauanga

By the end of this phase, students know that statistics is about learning from data and measuring, controlling, and communicating uncertainty. Sourcing data and learning from it are at the heart of evidence-based decision making. Data-based information is used to inform and influence decisions, behaviours, policies, and opinions. Data exploration requires a combination of data practices, such as transforming and visualising data. Drawing on contextual knowledge is integral to all statistical investigation and sense making. Statistical enquiry can be used to explore or understand a population, system, community, process, or issue and to inform action and advocacy.

Students learn that data can be extracted from a wide range of sources. How we collect data affects the applicability, quality, diversity, and quantity of the data, as well as the conclusions we draw from it. We need to take care about sources of bias. Data collection and use includes an ethical responsibility to obtain informed consent, to respect privacy, and to protect and benefit individuals and their environment.

Students know that, for statistical inference, data is sometimes obtained using random sampling to enable and justify sample-to-population inferences. Sometimes experiments are used to justify cause-and-effect causal inferences. Students use a range of simulation methods for different situations – for example, randomisation tests can be used with experiments to assess the strength of evidence for the existence of differences between treatments. Bootstrapping, for building confidence intervals, is a simulation method used to allow for uncertainties in estimation due to sampling error.

Students understand that there are situations in life that involve uncertainty. Probability can help us think about these situations and make decisions on them. Probability models can be developed from gathered data or from theory; both approaches make assumptions that might not be valid. Generating data from a probability model through simulation can demonstrate what outcomes are likely or unlikely under certain conditions, as well as the variability of the outcomes. Uniform, binomial, Poisson, and normal probability distributions have recognisable key features and can be used to model situations.

Students learn that findings from data are tentative and subject to revision when more evidence and insights become available. Communications with embedded statistical information should be critiqued using a statistical lens.

Do

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it in a systematic and organised way. They can identify relevant prior knowledge, conditions, assumptions, constraints, relationships, and concepts. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate relationships, objects, and ideas.

Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They make connections to ideas in other learning areas and with diverse cultural, linguistic, and historical contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others. They present reasoned, coherent explanations and arguments for an idea, solution, or process.

Phase 5 Years 11–13

Teaching sequences

Navigating pathways and developing agency to help shape the future
Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the fifth phase of learning at school.

Throughout phase 5, emphasise the importance of independent thought, intellectual rigour, and the ability to engage with complex, abstract mathematical and statistical concepts. Encourage students to approach problems critically and to explore advanced mathematical and statistical theories and applications. Motivate students to extend their thinking beyond standard procedures to find creative solutions. Facilitate an environment where they are expected to not only solve problems but also to explore the underlying principles of, and connections between, different areas of mathematics and statistics. Encourage them to critique mathematical and statistical arguments, identify strengths and weaknesses, and refine their reasoning. Support students' development of sophisticated mathematical and statistical communication skills, including the ability to present and defend their ideas clearly and persuasively. Empower them to take full ownership of their learning, and reinforce the expectation that they are all capable of engaging with challenging material. Provide opportunities for them to demonstrate their understanding in diverse and creative ways.

Explicit teaching

- Illustrate the steps involved in working through a sophisticated problem using worked examples. Support students with appropriate scaffolds to enable them to explore mathematical and statistical connections.
- Plan for students to actively recall learning, interleaving known concepts and procedures with new learning. Provide regular opportunities for practice to support the development of procedural fluency.
- Ensure students are given opportunities to apply their learning to unfamiliar contexts that help them see the connections between mathematics, statistics, and the wider world.

Positive relationships with mathematics and statistics

- Support students to persevere with challenging tasks. Help them to recognise the value of continued practice and the growth that comes from engaging deeply with mathematics and statistics.

Rich tasks

- When designing tasks, consider different contexts, levels of difficulty, time requirements, procedures to be used, and multiple entry and exit points. Use contextual situations that are familiar for students, and connect the mathematics and statistics strands and disciplines as much as possible. Carefully choose questions and statements that will help develop deep conceptual understandings. Include checks during lessons to give students opportunities to refocus, summarise ideas, and ask questions to clarify next steps.
- Support students to convert their wonderings or observations into conjectures or claims. Encourage them to check if their conjectures always work, if there are times when they don't work, and if they can be extended. Represent their working and reasoning using diagrams, materials and digital tools to engage them.
- Check in at specific points in investigations and support students to progress their work. Respond as appropriate with specific, targeted teaching of the mathematical or statistical skills needed to progress the investigation.
- Encourage students to try applying new skills and processes in their investigations. Support them to select the most appropriate process for solving a problem. Help them to recognise that if their first approach doesn't work, trying another approach is part of the richness of problem solving.
- Encourage students to be respectful of others' views. As part of rich tasks, encourage them to include different perspectives – for example, what is important to others in mathematics and statistics, and appropriate ways of investigating and collecting information and data about them.

Communication in mathematics and statistics

- Encourage students to collaborate and build on each other's thinking, learning, and arguments.
- Model the correct use of mathematical and statistical language, vocabulary, and notation. Support students to correctly use them, ensuring that they get multiple opportunities to practise using unfamiliar terms and notation.
- Empower students to reason mathematically and statistically. Help them to develop confidence in critiquing their own and others' mathematical conjectures, explanations, and justifications.

Mathematics and statistics, Year 11: Teaching sequence

Navigating pathways and developing agency to help shape the future
Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō

Number

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› recognise and use the properties of numbers, including those of: – natural numbers and integers (positive, zero, and negative) – prime, rational, and irrational numbers – reciprocals	**Connect** the properties of numbers with the use of numbers in other strands (e.g., measurement) and learning areas (e.g., science).
› calculate with numbers in scientific notation	**Connect** with other learning areas to explore how they use and write very large and very small numbers.
› use rounding and estimation to predict outcomes and to check the reasonableness of calculations	Demonstrate how estimating answers and the expected size of numbers is a way of checking the results of calculations for reasonableness. Support students to **justify** estimated solutions.
› round to the degree of precision required by the context	Apply rounding in practical situations and explain why it is important, discussing different contexts in which it occurs.
› perform operations on positive and negative rational numbers in any form (i.e., whole numbers, fractions, and decimals), including raising them to integer powers and taking roots of them, where defined	Prompt students to use known benchmarks, inverse operations, and knowledge of place value. **Represent** solutions using the most appropriate number form (e.g., a percentage, a ratio, scientific notation, or an exponent). **Investigate** numerical solutions in practical situations, (e.g., financial situations that involve rates). Explore common misconceptions and errors. For example, compare the effect of increasing a quantity by a percentage and decreasing the increased quantity by the same percentage.
› use fractions, decimals, and percentages, choosing whichever is most appropriate	
› apply rates and ratios, using proportional and inverse proportional reasoning when appropriate	
› perform operations with percentages, including increasing or decreasing a quantity using a single multiplier	

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› calculate compound interest over a fixed amount of time, compounding annually, quarterly, monthly, or daily.	Use practical examples of interest rates to **investigate** the outcomes of different investments or loans over time.

Algebra

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› find solutions that maximise or minimise a quantity while meeting the constraints of a situation, by making lists, tables, and graphs and comparing values	Use contextual situations, with an emphasis on checking that answers are appropriate for each situation. **Investigate** optimal solutions for maximising or minimising quantities.
› operate on numeric and algebraic expressions that have integer exponents, applying exponent rules	Explore common misconceptions and errors (e.g., the misconception that the distributive property works with powers and roots).
› add, subtract, multiply, and divide algebraic fractions with numeric denominators	**Connect** with fraction skills in the number strand.
› simplify, expand, and factorise algebraic expressions	Demonstrate the area method for expanding, and **connect** it with other approaches. **Generalise** number operations using algebraic representations for the commutative, associative, and distributive properties of numbers. **Investigate** the relationships between number properties and algebraic rules, to develop an understanding of why the rules work.
› substitute into, rearrange, and simplify algebra expressions or formulae	**Connect** with measurement through the use of formulae for perimeter, area, and volume.
› interpret, graph, and express linear relationships in any form	**Connect** a range of algebraic skills (e.g., forming, graphing, and solving) and explore how they work together. Use appropriate digital tools to explore the connections. Use a series of mathematical statements to **explain** how the solutions to equations or inequalities are reached.
› form and solve linear equations and inequalities	
› form and solve pairs of simultaneous linear equations with two variables, and give geometric and contextual interpretations	

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› recognise the relationships between elements of a linear or quadratic pattern, write an equation to represent the rule for the pattern, and use the equation to make conjectures	**Connect** with number skills and properties of graphs to help students understand the connection between rules, equations, and the properties. **Investigate** the relationships between the elements of patterns using words, formulae, and graphical **representations**.
› form and solve quadratic equations	**Explain** why there are different numbers of solutions for quadratic equations and the link between the solutions of quadratic equations and the graph of a parabola. Consider everyday situations in which a quadratic model could be useful. **Explain** the zero product property and the benefit of rearranging a quadratic equation to equal zero in order to find solutions.
› for quadratic and exponential functions with a positive integer base: – make a table and graph, given an equation – interpret key information and features of a graph.	**Connect** the graphing of quadratic and exponential functions, and their key features, with linear equations and graphs, noting similarities and differences. **Connect** the key features and graphing of quadratic and exponential functions with linear equations and graphs, noting similarities and differences. **Represent** exponential growth and decay using appropriate graphical forms.

Measurement and Geometry

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› estimate, calculate, and represent measurements accurately	Use a variety of analogue and digital measurement tools. Consolidate the skills of measuring using practical situations and examples in which students are doing physical measurements and working with non-theoretical situations. **Represent** measurements accurately, when both estimating and taking measurements, and use them in calculations. **Connect** the practical collection of data in measurement problems with the use of appropriate units, including those for rates.

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› connect significant figures and limits of accuracy with measurements	**Investigate** limits of accuracy for maximum and minimum solutions in problems from practical situations that round measurements to significant figures. **Justify** appropriate rounding and units in measurement solutions using significant figures and limits of accuracy.
› convert between metric units, including for volume, capacity, mass (weight), and derived units such as rates and ratios	**Connect** the units and conversions needed for different numbers of dimensions (e.g., m^3 to mm^3 will be × 1000^3).
› use and apply Pythagoras' theorem in 2D and 3D situations › apply trigonometric ratios (sine, cosine, and tangent) in right-angled triangles to find unknowns, in 2D and 3D situations	Use materials and diagrams to **represent** situations and **visualise** problems (e.g., signalling with colours to identify different parts of a diagram or problem). **Connect:** › the properties of 2D right-angled triangles illustrated by Pythagoras' theorem to investigations of right-angled triangles in 3D shapes › algebraic thinking to the solving of problems using trigonometric ratios in right-angled triangles.
› find the surface area and volume of 3D shapes composed of cylinders, pyramids, cones, spheres, or non-rectangular prisms	Use materials and diagrams to **represent:** › nets for the surface areas of 3D shapes › breakdowns of sections for composite shapes, to find their volumes. **Connect** with: › other strands (e.g., algebra, when using formulae for area and volume) › contextual situations, with an emphasis on checking that answers are appropriate for each situation.
› resize a shape by a factor and find the perimeter, area, or volume of the resulting shape, relating this to the scale factor	**Investigate** the effect of changes in dimension (e.g., from length to area) on scale factors, using practical measuring activities.
› apply transformations to lines and parabolas in the XY plane, relating changes in a graph to changes in its equation.	**Investigate** transformations of lines and parabolas in the XY plane. **Connect** the different forms of linear and quadratic equations to the key features of linear and parabolic graphs and to how the features change, depending on transformations of the graphs.

Statistics

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
Problem › investigate, using multivariate datasets from observational studies, summary, comparison, time-series, and relationship situations by: – posing an investigative question, in particular a summary or comparison question about a population of interest, or a time-series or relationship question about a group of interest – making conjectures or assertions about expected findings	**Connect** with students' interests and backgrounds and with other subjects to find contexts for meaningful investigations. Demonstrate how to pose an **investigative** question for situations involving multivariate datasets: › summary and comparison situations, including the variable and population of interest › relationship and times-series situations, including the variable and group of interest. Support students to discuss, **explain**, and **justify** their conjectures or assertions.
Plan › plan how to collect or source data to answer the investigative question, including: – identifying and justifying the variables needed to answer the question – planning how to make and justifying valid and reliable measures for the variables (when collecting data) or interrogating sourced datasets to understand what the variables measure and how they were measured – identifying the group or population of interest or (for sourced data) 'who' the data was collected from – determining if the investigative question is about a group or a population – if a population, determining what sample size is needed to make a decision about what is happening in the population – using interrogative questions to check that ethical practices are considered throughout the entire enquiry cycle	**Explain** and **justify** methods of data collection, 'who' to measure, and what and how to measure, in order to answer the investigative question. **Investigate:** › the impact of increasing the sample size on whether or not a call can be made about what is happening in the population (in comparison situations) › how people's different interpretations of instructions or use of devices can affect the quality of collected data. Closely examine secondary datasets to **explain** their variables and group(s) or population(s) of interest and the original investigator's plan. **Explain** how to apply ethical practices to data collection and use, including viewing data as taonga.

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
Data › collect or source data, including: – using random samples to collect or source data about a population or populations (for summary and comparison situations) – creating a data dictionary (for collected data) or identifying the metadata (for sourced data) – managing the data collection process, including considering possible errors and ethical concerns – storing the data, ensuring it is kept safe and confidential – sorting the data in preparation for analysis – checking the validity of data and, if appropriate, making simple edits and imputations (i.e., cleaning it) – recategorising or constructing new variables if needed to support answering the investigative question	Support students to use a variety of methods for collecting or sourcing the data, including: › making measurements › observing outcomes › using survey forms › using secondary data sources › using random samples. **Explain** and **justify:** › methods for recording and storing data › the use of recategorised variables and why new variables may have been constructed. **Investigate** how measurers, collection methods, and processing the data can introduce errors into the data. Support students to identify errors in the data, **connecting** to the context, **explaining** why they are errors, and, if appropriate, cleaning the data using appropriate techniques.
Analysis › create multiple data visualisations for the investigation, providing global and local views › describe and reason from the data visualisations, including: – identifying relevant features in distributions – interweaving the context in descriptions of distributions – making an informal inference about what might be happening in the population, based on visual considerations in comparison situations – drawing a line or curve of best fit to make conjectures about or predict possible y values (for the response variable) for given x values (for the explanatory variable) in relationship situations, and, where appropriate, indicating a possible (informal) interval for predictions, using visual methods – making informal predictions about group membership (e.g., in relation to socio-economic status or educational attainment)	**Represent** data using dot plots, bar graphs, frequency tables, box plots, histograms, paired plots, time-series graphs, two-way tables or graphs, and scatter plots. Check data visualisations for misleading features or information. Read the data, read 'between' the data, read 'beyond' the data and read 'behind' the data. **Explain** how different data visualisations have different features and how to describe them in context, using, for example: › fractions, proportions, and percentages › the mode, median, mean, and distributional shape › maximum and minimum values › the interquartile range, lower quartile, upper quartile, middle 50%, and range › descriptions such as more than, less than, at least, at most, between A and B › outliers, clusters, gaps, joint and conditional proportions › an eyeballed line or curve of best fit, seasonal patterns, and the long-term trend. *(continued on the next page)*

During year 11 *Informed by prior learning, teach students to:*	Teaching considerations
Analysis *(continued from the previous page)*	Describe what is seen and is not seen in data visualisations, recognising that data are numbers with context and that the context includes variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables. Compare data visualisations of the same variable for different groups, looking at similarities and differences. In comparison situations, make an informal inference about what might be happening in the population, using the 'Making the call Year 11' decision guide.
Conclusion › answer investigative question and justify and communicate findings by: – using evidence from analysis, with an awareness of variability – generalising beyond the sample to the population (for summary and comparison situations), providing evidence and accounting for uncertainty – providing explanations for observed patterns in the data – reflecting on conjectures or assertions – critically evaluating their investigation throughout all phases of the enquiry cycle – structuring their evidence and findings into a coherent whole that effectively communicates the entire statistical enquiry to a non-specialist	Have students practise answering the investigative question succinctly and in a way that a non-specialist would understand, using evidence from analysis and informal inferences or predictions, with an awareness of variability. Explore relevant explanations or interpretations of findings connected to the context of the situation, including the group or population of interest and the variable or variables of interest. Support students to evaluate and recommend any changes to the processes they used. Ask them to **explain** whether or not their findings align with their initial conjectures or assertions, if what was found makes sense with what is known about the situation, and if there are any limitations to the findings. Ask students to consider, as a result of the statistical investigation, what else they could **investigate**, starting the PPDAC cycle again.
Statistical literacy › evaluate others' statistical findings, reports, and data visualisations to see if the claims in them are reasonable and support or misrepresent the data or situation.	Explore existing statistical reports, graphs, visualisations, and claims in order to determine their validity. Use readily available media (e.g., news media or social media) as sources. Use interrogative questions to evaluate data-collection methods that involve, for example, ethical considerations, analysis processes, visualisations, and statements of findings.

Probability

During year 11 *Informed by prior learning, teach students to:*	Teaching considerations
› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by: – posing an investigative question – making conjectures about the distribution of outcomes – recognising the underlying structure (e.g., with or without replacement) – constructing a model using theoretical probabilities, probabilities based on past data, or conjectured probabilities – identifying model assumptions and limitations – deciding on and justifying the number of trials, the tools to be used, and the recording method – collecting and recording data, including by running simulations – creating data visualisations for the distribution of observed outcomes from the experiment and for all possible outcomes for theoretical probability models, where they exist – describing what these visualisations show – finding probability estimates for the different outcomes – proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists – identifying similarities and differences between their findings and those of others – reflecting on conjectured outcomes – identifying similarities and differences between findings from the probability experiment and associated theoretical probabilities, as appropriate	Conduct practical experiments in class involving materials and digital tools, collectively critiquing and improving plans and proposals. Compare theoretical probabilities with experimental probabilities and explore reasons for differences. **Investigate** every day chance-based situations, patterns in possible outcomes, theoretical and experimental distributions, and relative frequencies from data investigations. This could be using the statistical enquiry cycle. **Represent**: › probability outcomes (theoretical and experimental) using lists, tables, tree diagrams, tally charts, visualisations of distributions, words, numbers, and digital tools › probability models. **Explain** how to describe and use probability concepts – for example: › outcomes, events, and the sample space › trials, simulations, models, and theoretical and experimental probability › with and without replacement, independence and dependence, joint and conditional probabilities › the law of large numbers › probability estimates, theoretical probability, and probability distributions › chance, randomness, and variation. **Connect:** › anticipated distributions with theoretical distributions and experimental distributions, linking **representations** (e.g., two-way tables) with distributions, words, and numbers › observed data, models, and simulated data.
› identify why claims about probability experiments might not be valid, and consider improvements	Support students with critical evaluation and reasoned conclusions.

During year 11 *Informed by prior learning, teach students to:*	**Teaching considerations**
› engage in critical thinking for chance-based situations by: – recognising potential issues in the assumptions of a theoretical probability model that has been applied – evaluating the reasonableness of others' claims – explaining whether a claim is misleading and why and how probability may have been used in a particular way to arrive at the claim (e.g., in advertising).	**Connect** with the use and analysis of statistics in the media, to gain an overview of how statistics and probability are shared outside academia. Explore different interpretations of and reactions to the same experimental results. Use interrogative questions to evaluate others' findings, including their model assumptions, numbers of trials, visualisations, and statements.

The language of mathematics and statistics: Phase 5

	Year 11 *Students will know the following new words:*		
Number	› compound interest › irrational number	**Statistics**	› informal inference
Algebra	› exponential growth or decay › function › maximise › minimise › optimise › simultaneous equations	**Probability**	› two-way table
Measurement and Geometry	› limits of accuracy › non-rectangular prism		

Mathematics, Years 12–13: Teaching sequence

Navigating pathways and developing agency to help shape the future
Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› manipulate algebraic expressions, including algebraic fractions, by: – simplifying, rearranging, expanding, and factorising – adding, subtracting, multiplying, and dividing – raising them to integer and unit fraction exponents	› manipulate algebraic expressions, including those involving roots, by: – simplifying, rearranging, expanding, and factorising – adding, subtracting, multiplying, and dividing – rationalising the denominator – raising them to rational exponents	**Investigate** how combining manipulations can simplify a task (e.g., how factorising can enable the reduction of an algebraic fraction). **Connect** with logarithms and operations. Consider the connections between **generalising** number operations and algebraic manipulation. **Investigate** situations such as: › different ways of expressing the same expression or equation, and deciding which is most appropriate for a given scenario › the use of algebraic proofs.
› perform operations on logarithms of base b (b > 0, b ≠ 1), including the common logarithm b = 10: – simplify logarithms using number properties (e.g., $\log_5 125 = 3$, since $5^3 = 125$) – convert between logarithmic and exponential forms – use log rules for products, quotients, and powers – given data for x and y, linearise it by estimating parameters in relationships of the form $y = ax^n$ and $y = kb^x$	› perform operations on logarithms of base b (b > 0, b ≠ 1), including the natural logarithm b = e: – simplify logarithms using number properties – convert between logarithmic and exponential forms – develop and use log rules, including the change-of-base rule – linearise exponential data	**Connect** logarithms to exponential functions, emphasising the inverse relationship. Use graphical **representations** to explore these functions. **Investigate** situations such as: › how changes to functions affect their graphs › how the same change on an exponential graph and its inverse log graph affects the two graphs (e.g., what is the effect of +2 on each?) › how various scales work (e.g., for measuring the magnitude of earthquakes, or for measuring sound levels) › how different logarithmic scales can be used with graphs to linearise an exponential function.

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› form, solve, and graph quadratic relationships, equations, and inequalities with rational coefficients, including by completing the square and using the discriminant	› form, solve, and graph polynomial functions, equations, and inequalities with rational coefficients, including those functions with complex roots	Support students to use paper and digital tools to explore graphing functions and solving systems of equations. **Connect** features of graphs to the solutions of equations and inequations. **Investigate** the inverses of given functions. **Investigate** situations such as: › compound interest, calculated daily › completing the square to **explain** the development of the quadratic formula › the relationship between the nature of the discriminant and the number of roots in a quadratic function › Kirschhoff's laws in physics.
› form, solve, and graph exponential and logarithmic functions and equations, not including those with base *e*	› form, solve, and graph exponential and logarithmic functions and equations, including those with base *e*	
› form, solve, and graph systems of two simultaneous equations, one of which may be linear, in two dimensions, and interpret solutions	› find the optimal solutions of a system of linear inequalities (i.e., linear programming)	
› use algebra on the XY plane (i.e., coordinate geometry) to find measurements and develop geometric facts related to points, lines, and circles, including the equation of a circle	› use parametric equations to represent curves and circles	At year 12, **connect** algebra on the XY plane to other learning – for example, to calculate: › the distance between points, using Pythagoras' theorem › the midpoint of two points as the mean of the coordinates › intersections between lines and curves, using systems of equations › the approximate slope of a tangent line, using secant lines (i.e., using two points to find the slope). At year 12, **investigate** situations such as: › deciding on the nature of a triangle using distances between points and gradients › showing the relationship between the distance formula and the equation of a circle › using computer-aided design (CAD) for architecture and design, referenced from the point (0,0) on an XY graph. At year 13, **investigate** time in **connection** with parametric equations (e.g., how graphical representations change with changes to time).

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› use the binomial expansion of $(a + b)^n$ where n is a positive integer, including finding the binomial coefficients using the combinations 'n choose r' and Pascal's triangle	› perform operations on complex numbers in the Cartesian form $z = a + bi$: – find the real part, imaginary part, conjugate, modulus, and argument – calculate sums, differences, products, and quotients – use and draw Argand diagrams	At year 12, **investigate** situations such as: › the multiplication principle and how this leads to combinations and Pascal's triangle › how using the binomial expansion for expanding powers of expressions simplifies the process. At year 13, **investigate** generating fractals with complex numbers (e.g., the Mandelbrot set).
› work with sequences, including those that are arithmetic, geometric, or defined recursively, and series, with and without the use of sigma notation	› manipulate and apply complex numbers: – find the modulus, r, and argument, θ – convert between Cartesian form, the polar form $z = r(\cos\theta + i \sin\theta)$, and the exponential form $z = r e^{i\theta}$ – calculate products, quotients, roots, and integer powers in polar and exponential forms – geometrically interpret addition, subtraction, and multiplication	At year 12, **connect** sequences and series with linear and quadratic patterns, and develop formulae so students are able to **explain** and **justify** the formulae for different sequences and series. At year 12, **represent** and generate sequences using spreadsheets and graphing applications. At year 12, **investigate** situations such as: › the spread of disease › salary increases and decreases › population growth › charging and discharging capacitors in physics. At year 13, **investigate** situations that use complex numbers – for example: › solving the same equation in Cartesian, polar, and exponential forms, especially in situations involving powers of complex numbers › electromagnetic waves and electric currents › option pricing models in finance › medical imaging such as MRI (magnetic resonance imaging).

During year 12 *Informed by prior learning, teach students to:*	During year 13 *Informed by prior learning, teach students to:*	Teaching considerations
› use the properties of circles to: – identify angles and convert between radians and degrees – calculate the area of a sector and length of an arc – identify that $\cos^2\theta + \sin^2\theta = 1$ for all values of θ – develop, using a unit circle, the sine and cosine functions for all real numbers and their graphs, noting symmetries and lengths of periods – use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to graph the tangent function	› form, solve, and graph sine, cosine, and tangent functions and equations	**Connect** $\cos^2\theta + \sin^2\theta = 1$ with Pythagoras' theorem and the equation of a circle. **Investigate** situations such as: › how the graphs of f(x) = sin(x) and f(x) = cos(x) can be constructed from the unit circle › real-life cyclical relationships (e.g., the movement of a Ferris wheel, the motion of tides, or simple harmonic motion).
› develop and use the sine rule, cosine rule, and area formula $\frac{1}{2}ab\sin C$ to find the side lengths, angles, and area of a triangle	› use trigonometric identities to simplify and rearrange trigonometric expressions	At year 12, **investigate** situations such as: › the area of a non-rectangular section of land › the volume of a mud slide, calculated using cross sections › the ambiguous case of the sine rule. At year 13, **investigate** expressions involving compound angles, double angles, products, and sums in situations that involve calculations (e.g., finding exact values or integrals).

During year 12 *Informed by prior learning, teach students to:*	During year 13 *Informed by prior learning, teach students to:*	Teaching considerations
› perform operations on polynomial (constant, linear, quadratic, and cubic), power (where the power is an integer), square root, exponential, common log, absolute value, and (in degrees) sine, cosine, and tangent functions: – identify the domain and range – identify key features of graphs – use function notation – transform the graph of $y = f(x)$ to $y = Af(x - B) + C$ – interpret algebraic solutions of equations graphically	› perform operations on power, exponential, natural log, absolute value, sine, cosine, tangent, and piecewise functions: – identify the domain and range – identify key features of graphs – use function notation – transform the graph of $y = f(x)$ to $y = Af(Bx - C) + D$, where B = 1 or C = 0 – interpret algebraic solutions of equations graphically – form and use composite functions and notation – find inverse functions, restricting the domain if needed	**Investigate** power functions of the form ax^n where n is an integer (at year 12) or a rational number, including square roots (at year 13). Demonstrate how naturally occurring curves can be modelled and how the models can be used to explore aspects of practical situations, including making conjectures. **Generalise** transformations of graphs to enable comparisons across different functions. **Investigate** situations such as: › finding the best function to model a given natural curve, exploring how domain and range can influence the choice of the function › how functions can be used as models in applications such as computer-aided design (CAD) › supply and demand curves in economics › height from the ground and speed in skydiving, before and after opening one's parachute › real-life cyclical relationships that use trigonometric relationships to solve contextual problems.
	› determine where a function is continuous and differentiable, identify discontinuities, and estimate limits from a table or graph	**Connect** the continuity of a function with its graph and equation. **Investigate** situations such as: › traffic flow in towns or cities › changes in the stock market.
› for polynomial and integer power functions: – sketch the graph of the gradient function from the graph of the original function – sketch the graph of a function from the graph of its gradient function	› sketch a graph of the derivative function from the graph of a function that is differentiable over the given domain, and sketch the graph of a function from the graph of the derivative function	Use paper and digital tools to explore key features of the graphs of a function and its derivative function. Demonstrate through the use of appropriate tools the connection between the procedural (polynomial differentiation rule) and conceptual approaches for finding the gradient at a point on a curve. **Investigate** situations such as: › the family of possible functions for a given gradient function › the relationship between the order of a polynomial function and the order of its gradient function › how the key features of a gradient function relate to the key features of its original function, and vice versa.

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› apply features of constant, linear, quadratic, cubic, and integer power functions: – find the gradients and equations of secant lines – use the relationship between the gradient of a graph and tangents at points as an approach to differentiation at a point – identify significant features by using gradients, including intervals of increase and decrease, stationary points, maxima, and minima	› differentiate from first principles constant, linear, and quadratic relationships	Draw tangent lines to explore features of graphs. **Investigate** situations such as the rates of change of the gradients for a range of functions and the relationships between the features of these functions.
› perform operations on polynomial and integer power functions to: – find the derivative function – find the gradient and equations of tangent lines at a point – find points where the gradient has a particular value – identify significant features by using the derivative, including intervals of increase and decrease, stationary points, and local and absolute extrema	› perform operations on constant, rational power, exponential (base e), natural log, sine, cosine, and tangent functions – and functions created from them by sum, difference, product, quotient, and composition – to: – find the derivative function – find the gradient and equations of tangent and normal lines at a point – find points where the gradient has a particular value – identify significant features by using the derivative, including intervals of increase and decrease, stationary points, and local and absolute extrema	Demonstrate systematic approaches to evaluating the characteristics of functions (e.g., their gradients). **Investigate** situations such as: › the use of kinematics in practical situations › the use of calculus in economics (e.g., in profit, marginal cost analysis, and price elasticity (tangent and normal) calculations) › the efficient use of materials (e.g., in packaging).
	› find and use higher-order derivatives, including using the second derivative to determine concavity, local extrema, and points of inflection of the graph of the function	

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
	› calculate indefinite integrals for: – rational power, exponential base e, sine, and cosine functions – functions of the form $f(g(x))\,g'(x)$ – functions created from those above by sums, differences, and constant multiples	Demonstrate and **explain** systematic approaches to using numerical methods. **Investigate** situations such as: › areas of irregular shape in nature that can be modelled using functions › the **connections** between anti-derivatives, definite integrals, and area.
› use numerical methods on $f(x) > 0$ to find the area between a curve and the x-axis, using rectangles.	› calculate definite integrals: – using numerical methods, including Riemann sums – using the fundamental theorem of calculus	
	› form and solve simple differential equations (e.g., with a boundary condition to determine the constant term).	**Investigate** situations such as: › population growth › radioactive decay › Newton's law of cooling.
› use mathematical modelling to approximate applied situations	› use mathematical modelling to approximate applied situations	**Connect** year-level mathematical concepts, skills, and processes with meaningful contexts. **Explain** that modelling is open-ended and messy and involves connections between many different skills. When students are modelling, they must be making genuine choices. **Explain** and **justify** limitations with models, including why some values may not work. **Investigate** situations such as: › estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed › planning a table tennis tournament for 7 players at a club with 4 tables and in which each player will play against every other player › designing the layout of the stalls at a school fair so as to raise as much money as possible › analysing stopping distances for a car › modelling a changing savings account balance, investment growth, bacterial colony growth, or medication absorption.

The language of mathematics: Phase 5

Year 12 *Students will know the following new words:*		Year 13 *Students will know the following new words:*	
› absolute or local extrema › absolute value › arc › arithmetic, geometric › binomial › derivative function › differentiation › discriminant › domain, range › linearise › logarithm	› period or periodic › polynomial function › radians › recursive › secant line › series › sigma notation › sine, cosine, tangent › stationary point › tangent line › trigonometry, trigonometric	› Argand diagram › argument › Cartesian, polar, or exponential form › complex root › complex, real, or imaginary number › concavity › conjugate › continuous, continuity, discontinuous, discontinuity › definite or indefinite integral › differentiable	› differential equations › modulus › natural log › normal line › parametric equation › piecewise function › point of inflection › rationalise › recurrence interval › trigonometric identity

Statistics, Years 12–13: Teaching sequence

Navigating pathways and developing agency to help shape the future
Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō

During year 12 *Informed by prior learning, teach students to:*	During year 13 *Informed by prior learning, teach students to:*	Teaching considerations
› identify opportunities for using and exploring data to understand and learn about situations	› identify opportunities for using and exploring data to understand and learn about situations, and the possible impact of doing so	Demonstrate how to explore data-generating situations and how to identify where data can be used to solve a problem. **Investigate** statistical or chance-based situations in order to deepen understanding of them. Consider: › the possible impacts of a situation and whether or not data would help to understand and explain them › the possible impacts and benefits of potential findings or some research work › the **connection** between personal interest and the wider impacts of a situation. **Explain** and **justify** how thinking about 'what if …' and 'why should anyone care …' can inform a situation. Integrate relevant contextual knowledge and alternative perspectives.
› construct data from sources such as text, images, sounds, and movements (including in space and time)	› construct, manipulate, and restructure data from sources such as text, images, sounds, and movements (including in space and time)	Use data technologies to: › construct data from a variety of non-traditional sources › **connect** representations › manipulate and restructure data. Define, **explain**, and **justify** variables and how they will be measured (e.g., perceptions of things such as pain levels).

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› design, pilot, and refine data-collection instruments (e.g., questionnaires, sensors, and experiments) for the purpose of exploring or understanding a system, community, process, or issue	› design, using experimental design principles, and carry out an experiment for the purpose of identifying whether a causal relationship exists	Demonstrate how to design, pilot, and (at year 13) refine questionnaires for collecting data. Explore survey or data-collection questions, considering how question design impacts on the data that is collected. Define, **explain**, and **justify** measures for variables to ensure they will capture the data required. At year 12, show how to design and pilot experiments for collecting data. **Investigate** methods and purposes for data collection – for example: › selecting appropriate instruments for collecting data, such as sensors › designing and conducting experiments to collect data (at year 12) › undertaking experiments involving a comparison of two independent groups to identify a causal relationship (at year 13).
› use ethical and responsible data practices and consider how decisions on collecting and generating data (including the questions asked and how measures are defined) will affect the applicability, quality, diversity, and quantity of the data	› use ethical and responsible data practices and consider how decisions on collecting and generating data will affect the applicability, quality, diversity, and quantity of the data	**Investigate** data-collection tools and how they are designed to ensure the privacy and safety of participants and do no harm. **Explain** and **justify** ethical decisions about data-collection processes – for example, those that involve: › investigative questions › survey questions › variables to be measured › a population. Give examples of how to: › check for bias in data-collection processes › evaluate others' data practices › explore different methods of sampling and the impacts they may have.

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› as part of exploring data: – develop new variables – create a range of data visualisations for different variables or combinations of variables – communicate what has been learned, drawing on key features, patterns, or trends in the data	› as part of exploring data: – merge data sources – develop new variables – create a wide range of data visualisations for different variables or combinations of variables – communicate what has been learned, drawing on key features, patterns, or trends in the data	**Investigate** how to explore and manipulate data – for example: › using different methods for cleaning data › developing new variables from existing information in the dataset › merging data tables (e.g., from different sources) (at year 13) › exploring visualising the data using a variety of tools, including manual and digital tools. **Investigate** patterns and trends, connecting to the data context and undertaking further exploration. Explain how to record what is seen during exploration, communicating what has been learned about the data context.
› calculate and interpret absolute risk and relative risk for hazardous situations, using two-way tables of counts and performing calculations to verify claims based on risks	› calculate risks for hazardous situations and make and evaluate communications on matters such as absolute risk, relative risk, increased or decreased risk, and recurrence intervals	Show how to use tools such as two-way tables to calculate absolute and relative risks. **Investigate** and evaluate absolute and relative risk statements in media and other communications. **Connect** to other ways in which the word 'risk' is used, especially in the field of risk management. Use a range of frequency-based **representations** of risk data to help in the interpretation and communication of absolute and relative risks. At year 13, **investigate** and **explain** risk management for events of low probability and high impact. **Explain** and **justify** considerations such as: › the influence of base rates › how recurrence intervals describe a risk situation and the misconceptions they can cause › alternative ways of communicating recurrence intervals to others › whether it is better to use absolute risks, relative risks, or both for comparisons in a given situation › the interpretation of recurrence intervals.

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› make sample-to-population inferences by generating and interpreting informal confidence intervals for medians and proportions on data visualisations	› make sample-to-population inferences by generating and interpreting formal confidence intervals for means, medians, and proportions, including differences, using the bootstrap simulation-based method	**Investigate** and use sampling variation: › for generating and interpreting medians, means, and proportions › to understand informal confidence intervals for medians and proportions (at year 12). At year 12, use informal confidence intervals to make sample-to-population inferences when comparing medians and proportions on data visualisations. **Investigate** situations such as: › how to interpret confidence intervals in context › how to **connect** confidence intervals for proportions with the rules-of-thumb for margin of error. At year 13, **investigate**: › the modelling ideas that underpin bootstrap confidence intervals › using the bootstrap simulation-based method to generate confidence intervals › how formal confidence intervals behave.
› conduct simulation-based tests for a single proportion to assess the strength of evidence	› make experiment-to-causation inferences involving a comparison of two independent groups by considering the study design and using a simulation-based randomisation test to assess the strength of evidence	At year 12, **investigate** situations such as: › when a test can be conducted to see how an observed proportion compares with a just-chance model › generating simulated data under a just-chance model, and using the results to assess if the observed proportion is compatible with the just-chance model › using a simulation-based test for a single proportion to test for a change in before-and-after data. At year 13, **explain** and **justify**: › why random allocation to treatment groups is important › how randomising group labels helps assess the plausibility of the 'chance-alone' explanation › how to construct and use randomisation tests to assess strength of evidence in context (using the difference between two means or two medians). At year 13, **connect** randomisation test results with appropriate language to communicate findings to others.

During year 12 *Informed by prior learning, teach students to:*	**During year 13** *Informed by prior learning, teach students to:*	**Teaching considerations**
› identify likely and unlikely outcomes within distributions, and explore the impact of conditioning on other variables, from the perspective of prediction	› develop, use, and evaluate statistical models, including regression, time-series, and classification models, to make predictions	At year 12, **connect** likely and unlikely outcomes with low-probability and high-probability outcomes, for distributions of numeric and categorical variables. **Investigate:** › whether the distribution of a variable of interest (numeric or categorical) changes depending on the value of an additional categorical variable, and whether this is a useful predictor (at year 12) › informal approaches to making predictions (at year 12) › ways of including information on a third variable to check if it adds predictive information › whether the values of a numeric variable of interest change depending on the value of an additional numeric variable, and (at year 13) whether an additional numeric variable is a useful predictor. **Investigate** and compare data-analysis and prediction situations. At year 13, identify potential practical consequences when predictions are incorrect. At year 13, **explain** ways of evaluating: › issues with the applicability of predictions › the predictive accuracy of models, using representations such as confusion matrices.
› communicate findings based on statistical evidence, including – the context of the situation – limitations – recommendations – potential biases (e.g., subjectiveness)	› communicate findings based on statistical evidence, including – the context of the situation – assumptions and limitations – impacts of potential decisions and models – recommendations – new conjectures – uncertainties – potential biases (e.g., subjectiveness)	**Connect** new understandings gained from data about the context of a situation to findings of primary importance when answering the investigative question. **Explain** and **justify** the strength and trustworthiness of answers to the investigative question by showing how assessment of limitations, including acknowledging potential biases (at year 12) and uncertainty (at year 13), supports the findings. Demonstrate how the impact of findings can be used for: › practical actions relevant to the context of the situation › generating future investigations.

During year 12 *Informed by prior learning, teach students to:*	During year 13 *Informed by prior learning, teach students to:*	Teaching considerations
› use representations such as probability tree diagrams and two-way tables to determine the probabilities of events, including joint, marginal, and informal conditional probabilities	› determine the probabilities of events using a combination of simple probability models, probability distribution models, and approaches such as conditioning and partitioning	**Represent** information about chance-based situations using visual probability tools. Use a range of representations to determine probabilities, including two-way tables, probability-tree diagrams (at year 12), and bar graphs, eikosograms, pachinkograms, and Venn diagrams (at year 13). At year 13, **explain** how to: › reverse the order of conditioning (e.g., in screening or diagnostic tests) › use partitioning when solving probability problems.
› use probability distributions (including normal, binomial, and uniform) as models and to identify likely and unlikely outcomes	› select and use models of probability distributions (including Poisson, normal, binomial, and uniform) to calculate probability estimates, recognising the underlying structure and conditions of the data-generating process	Explore the underlying structures of formal probability distributions to justify the best model for a situation. **Investigate** simulation and exact-calculation digital tools for obtaining probabilities. At year 13: › **explain** how a probability model can generate random data from a probability distribution › **connect** binomial and Poisson probability distribution models to relevant risk situations.
› calculate and interpret expected counts for simple probability models	› calculate and interpret expected values and standard deviations of discrete random variables	At year 12, use manual and digital tools to explore expected counts in context. At year 13, use an appropriate probability model for a discrete random variable, where the distribution is presented as a table of values with their probabilities, to calculate its expected value. Use formulae or digital tools to: › calculate and interpret expected values and standard deviations for known probability distributions (at year 13) › **investigate** standard deviation, as a measure of variation (at year 13).
› conduct large-scale, technology-supported simulations to estimate model probabilities	› use visual, technology-based informal tests to assess the goodness-of-fit of probability distribution models by comparing observed data with model-generated data	Use digital tools to **investigate** year-appropriate models for estimating model probabilities. Explain occasion-to-occasion or experimenter-to-experimenter variation in estimated probabilities, and the effect of sample size on this variation.

During year 12 *Informed by prior learning, teach students to:*	During year 13 *Informed by prior learning, teach students to:*	Teaching considerations
› explore changing the features of probability and probability distribution models and discuss the effects	› link the parameters of probability distributions to the distribution of probabilities, and discuss the effects of changing the parameters	At year 12, use an appropriate interactive digital tool to explore what distributions look like and to **connect** how the distribution of probabilities changes with changing parameter values. At year 13, **investigate** the effects of changing parameters on probability distributions.
› interpret and critique data- and model-based information and practices, data visualisations, and claims, from a variety of sources.	› interpret and critique data and model- based information, visualisations, and practices, embedded statistical information in spoken and written texts, and claims from a variety of sources.	At year 12, show how to use a set of critical questions to evaluate data- and model-based information, visualisations, and practices in the media. Critique the language and statistical information used by others when communicating claims, including: › identifying who is doing the research › checking for hidden agendas › critically evaluating a wide range of data- and model-based information in the media and from other sources (at year 13) › considering decisions about data-based information and the consequences of these decisions (at year 13) › appreciating how statistics are used to influence change for good and bad › recognising written and spoken communications that have embedded statistical information (at year 13) › interpreting informal margin of error based on poll-based reports for single proportions (at year 12) or for differences between proportions (at year 13). **Investigate** and explain the pervasiveness of digital data collection (e.g., through interactions with social media, GPS devices, or sensors) and algorithmic model-based products (e.g., advert recommendations online or risk predictions), along with the consequences of algorithmic bias.

The language of statistics: Phase 5

Year 12 *Students will know the following new words:*	Year 13 *Students will know the following new words:*
› absolute or relative risk › expected counts › informal inference › informal, joint, or marginal conditional probability › normal distribution › uniform distribution	› bootstrap › conditioning › discrete random variable › expected value › formal inference › margin of error › parameter (of a probability distribution) › Poisson distribution › randomisation test › regression › standard deviation